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Constrained point-to-point iterative learning control with experimental verification

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ABSTRACT

Iterative learning control is a methodology applicable to systems which repeatedly track a specified reference trajectory defined over a finite time duration. Here the methodology is instead applied to the point-to-point motion control problem in which the output is only specified at a subset of time instants. The iterative learning framework is expanded to address this case, and conditions for convergence to zero point-to-point tracking error are derived. It is shown how the extra design freedom the point-to-point set-up brings allows additional input, output and state constraints to be simultaneously addressed, hence providing a powerful design framework of wide practical utility. Experimental results confirm the performance and accuracy that can be achieved, and the improvements gained over the standard ILC framework.

1. Introduction

Iterative Learning Control (ILC) is a technique for controlling systems operating in a repetitive mode with the requirement that a reference trajectory $y_d(t)$ defined over a finite interval $0 \leq t \leq T$ is followed to a high precision. Examples of applications include control of wafer scanners (Heertjes & van de Molengraft, 2009), internal combustion engines (Heinzen, Gillella, & Sun, 2011) and gantry robots (Hladowski et al., 2010). Despite substantial developments over the last 25 years, little attention has been paid to the case in which the repeated operation may consist of a more general objective, which need not comprise the tracking of a static pre-defined reference trajectory (see Ahn, Chen, & Moore, 2007; Bristow, Tharayil, & Alleyne, 2006 for recent overviews of the literature). A small number of ILC schemes do not use a static reference, but these are associated with specific applications, such as gas metal arc welding (Moore & Mathews, 1997), underwater robotics (Kawamura & Sakagami, 2002) or liquid slosh in a packaging machine (Grundelius & Bernhardsson, 1999).

In many applications, however, the goal is to repeatedly follow a motion profile in which the error is only critical at certain points. Examples include production line automation, crane control, satellite positioning, robotic 'pick and place' tasks, and motion control within a stroke rehabilitation context (Freeman et al., 2009). A commonly applied technique for such tasks is point-to-point motion control, in which the objective is to ensure that, at a finite set of prescribed time instants, the system output equals a corresponding set of desired values. Point-to-point control strategies typically involve the generation of a suitable motion profile in advance, and then the design of a controller to track it. The most common approach to generating such a profile is Input Shaping which has been applied in a wide variety of ways (Dharne & Jayasuriya, 2007; Singh & Singhose, 2002; Dijkstra et al., 2000), although other approaches have also been used (see Belts, 1998; Doyle, 1995 and references therein).

The application of ILC in the area of point-to-point motion control offers the potential to benefit from the ability to learn from experience gained over previous trials of the task. The problem can clearly be tackled in the standard ILC framework by using any reference which connects the desired points. However, extra freedom is gained by removing the unnecessary constraint that the plant follow a pre-defined output between points, which can be exploited to increase performance. Several cases exist in which ILC has been applied to point-to-point motion control. van de Wijdeven and Bosgra (2008) used Hankel ILC to suppress residual vibrations, where controller matrices are determined through command shaping. An iterative learning scheme is again employed in Park, Chang, Park, and Lee (2006) for vibration suppression, and the control parameters are updated via an input shaping technique. In Ding and Wu (2007) a standard ILC controller is first applied but the control gains are chosen to minimize an error norm motivated by the point-to-point positioning operation rather than the tracking error along the task. These applications of ILC to point-to-point tracking involve the design of an initial reference, which is then static over all trials. Furthermore, they consider only a movement between points. An exception to this is the frequency-domain approach of Freeman,

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Cai, Rogers, and Lewin (2010) which also requires a predetermined reference trajectory defined over the whole trial duration, but modifies it after each trial, enabling increased convergence and robustness properties compared with using a static trajectory. This approach can also deal with multiple point-to-point movements. However, like the former methods, its performance depends on the initial predetermined reference trajectory which, being based on the nominal plant model, is not optimal with respect to the actual plant.

In contrast to the methodologies currently employed within ILC, this paper introduces an approach to the point-to-point problem which dispenses of a reference defined over the entire trial interval, thereby simplifying design and removing unnecessary constraints. Furthermore, for the first time it allows a general class of additional objectives, such as minimising the input energy or output velocity, to be specified by the designer in response to practical considerations and desired performance. This is not possible using the ILC approaches previously formulated. Through use of data collected over repeated executions of the task, these objectives, together with the point-to-point tracking task, can then be achieved robustly in the ILC framework.

The paper is organised as follows: Section 2 introduces the control problem and describes the plant, Section 3 applies the gradient descent approach to the point-to-point problem in the ILC framework, and then introduces additional constraints to be satisfied. In Section 4 Newton method based ILC laws are developed, using results derived in the previous section. Experimental results are presented in Section 5, and conclusions are given in Section 6.

2. Problem formulation

Let $\mathbb{R}$ denote the set of real numbers, with the norm of any column vector $x \in \mathbb{R}^n$ given by $\|x\|_2 = \sqrt{x^T x}$. Also $\rho(X)$ denotes the spectral radius of matrix $X \in \mathbb{R}^{n\times n}$, that is, $\rho(X) = \max_{1\le i\le n}|\lambda_i|$, where λ_i is an eigenvalue of X. The maximum and minimum singular values of X are denoted $\overline{\sigma}(X)$ and $\underline{\sigma}(X)$ respectively.

Consider the linear time-varying (LTV) discrete-time system:

$$x(i+1) = A(i)x(i) + B(i)u(i), \quad i = 0,1,\ldots,N-1$$

$$y(i) = C(i)x(i) + D(i)u(i), \quad x(0) = x_0 \tag{1}$$

defined over the trial length of N samples. Here $u(\cdot) \in \mathbb{R}^m$, $y(\cdot) \in \mathbb{R}^p$, $x(\cdot) \in \mathbb{R}^n$ with the state-space matrices $A(\cdot) \in \mathbb{R}^{n\times n}$, $B(\cdot) \in \mathbb{R}^{n\times m}$, $C(\cdot) \in \mathbb{R}^{p\times n}$ and $D(\cdot) \in \mathbb{R}^{p\times m}$. Over the trial duration the input and output sequences are given respectively by

$$u = [u(0)^T, u(1)^T, \ldots, u(N-1)^T]^T \in \mathbb{R}^{mN}$$

and

$$y = [y(0)^T, y(1)^T, \ldots, y(N-1)^T]^T \in \mathbb{R}^{pN}$$

Over each trial the relationship between the input and output time-series can be expressed by the system:

$$y = Gu + y_0 \tag{2}$$

where

$$G = \begin{bmatrix} D(0) & 0 & \cdots & 0 \\ C(1)B(0) & D(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C(N-1)(\prod_{i=1}^{N-2} A(i))B(0) & C(N-1)(\prod_{i=2}^{N-2} A(i))B(1) & \cdots & D(N-1) \end{bmatrix} \in \mathbb{R}^{pN\times mN} \tag{3}$$

Here

$$y_0 = \left[C(0)^T, \ (C(1)A(0))^T, \ \cdots, \ \left(C(N-1)\left(\prod_{i=0}^{N-2} A(i) \right) \right)^T \right]^T x_0 \quad \in \mathbb{R}^{pN}$$

is the response to initial conditions whose effect can be absorbed into the reference trajectory, so that without loss of generality it is assumed $y_0 = 0$, or equivalently $x_0 = 0$.

2.1. Standard ILC

The goal of the standard ILC framework is to construct a series of inputs which drive the system to track a reference:

$$y_d = [y_d(0)^T, y_d(1)^T, \ldots, y_d(N-1)^T]^T \quad \in \mathbb{R}^{pN}. \tag{4}$$

Let u_k and y_k be the input and output vectors respectively on the kth trial, with the tracking error $y_d - y_k$. The ILC problem is to find a sequence of control inputs satisfying:

$$\lim_{k\to\infty} \|y_d - y_k\|_2 = 0, \quad \lim_{k\to\infty} \|u_k - u_d\|_2 = 0$$

where u_d is the ideal control input.

2.2. Point-to-point ILC

Let the plant output be specified at a fixed number, $M \le N$, of sample instants given by $0 \le N_1 < N_2 \cdots < N_{M-1} < N_M < N$. Let the respective prescribed values of the output at these instants be $y_1, y_2, \ldots, y_M$, where $y_i \in \mathbb{R}^p$. The point-to-point ILC problem is to find a sequence of control inputs satisfying

$$\lim_{k\to\infty} \|\Gamma - \Phi y_k\|_2 = 0, \quad \lim_{k\to\infty} \|u_k - u_d\|_2 = 0 \tag{5}$$

where

$$\Gamma = [y_1^T \ y_2^T \ \cdots \ y_M^T]^T \quad \in \mathbb{R}^{pM}. \tag{6}$$

The $pM \times pN$ matrix Φ extracts elements of y_k that correspond to the prescribed point-to-point time instants, maintaining their order. To do this it has block-wise components given by

$$\begin{bmatrix} \Phi_{(i-1)p+1,(j-1)p+1} & \cdots & \Phi_{(i-1)p+1,jp} \\ \vdots & \ddots & \vdots \\ \Phi_{ip,(j-1)p+1} & \cdots & \Phi_{ip,jp} \end{bmatrix} := \begin{cases} I_p, & j = N_i + 1, \ i = 1,2\ldots M \\ 0_p & \text{otherwise} \end{cases} \tag{7}$$

in which I_p and 0_p are the $p \times p$ identity and zero matrices respectively.

The point-to-point formulation hence replaces the need for a reference defined at every sample, but corresponds to the standard ILC problem of Section 2.1 when $M = N$ and $\Gamma = y_d$ in which case $\Phi = I_{pN}$.

3. Gradient descent ILC

Point-to-point ILC can be considered an iterative numerical solution to the problem

$$\min_u f_1(u), \quad f_1(u) := \|\Gamma - \Phi G u\|_2^2 \tag{8}$$

The gradient descent method is a nonlinear minimization technique (Burden & Faires, 2001) which can be applied to solve this iteratively with the update

$$u_{k+1} = u_k - \frac{\beta}{2} \nabla_u f_1(u_k) = u_k + \beta(\Phi G)^T(\Gamma - \Phi G u_k) \tag{9}$$

where β is a positive scalar gain. In the ILC framework the experimentally obtained tracking error must be embedded into