Computers and Mathematics with Applications 64 (2012) 3210–3216

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Iterative learning control with initial state learning for fractional order nonlinear systems☆

Yong-Hong Lan *

School of Information Engineering, Xiangtan University, Xiangtan, Hunan 411105, PR China
Key Laboratory of Intelligent Computing and Information Processing of Ministry of Education, Xiangtan University, Hunan 411105, PR China

ARTICLE INFO

Keywords:
Fractional order
Iterative learning control
Generalized Gronwall inequality
Convergent

ABSTRACT

This paper presents a P-type iterative learning control (ILC) scheme with initial state learning for a class of α ($0 \leq \alpha < 1$) fractional-order nonlinear systems. By introducing the λ-norm and using a generalized Gronwall inequality, the sufficient condition for the robust convergence of the tracking errors with respect to initial positioning errors under P-type ILC is obtained. Based on this convergence condition, the learning gain of the initial learning and input learning updating law can be determined. Unlike the existing methods, the ILC scheme will not fix the initial value on the expected condition at the beginning of each iteration. Finally, the validity of the methods are verified by a numerical example.

1. Introduction

Fractional differential calculus dates back from the 17th century, but only until the recent decade was it applied to physics and engineering [1]. It was found that many systems in interdisciplinary fields could be elegantly described with the help of fractional derivatives and integrals, such as viscoelastic systems, dielectric polarization, electrode–electrolyte polarization and electromagnetic waves [2]. Furthermore, many fractional order controllers have so far been implemented to enhance the robustness and the performance of the control systems [3–5].

Iterative learning control (ILC) is one of the most active fields in control theories. The objective of ILC is to determine a control input iteratively, resulting in the plant's ability to track the given reference signal or the output trajectory over a fixed time interval. Owing to its simplicity and effectiveness, ILC has been found to be a good alternative in many areas and applications, e.g., see recent surveys [6–8] for detailed results.

In recent years, the application of ILC to the fractional-order system has become a new topic [3,9–12]. The authors in [9] were the first to propose the $\mathcal{D}^{\alpha}$-type ILC algorithm in frequency domain. In [10], the asymptotic stability of P$\mathcal{D}^{\alpha}$-type ILC for fractional-order linear time invariant(LTI) systems was investigated. The convergence condition of open-loop P-type ILC for fractional-order nonlinear systems was tried in [11]. Moreover, the $\mathcal{D}^{\alpha}$-type ILC for fractional-order LTI systems was discussed in [3,12]. However, in the above methods, the ILC algorithm must fix the initial value on the expected condition at the beginning of each iteration.

Motivated by the above mentioned research to the tracking problem of fractional-order systems, the open-loop and closed-loop P-type ILC updating law with initial state learning are applied to a class of fractional-order nonlinear systems. A sufficient condition for the robust convergence of the tracking errors under the proposed P-type ILC is proved. Based on this convergence condition the learning gains can be determined.

☆ This work was supported in part by the National Natural Science Foundation of China (61104072).
* Correspondence to: School of Information Engineering, Xiangtan University, Xiangtan, Hunan 411105, PR China.
E-mail address: lyhcsu@yahoo.com.cn.

0898-1221/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2012.03.086

The rest of this paper is organized as follows. In Section 2, the problem formulation and some preliminaries are presented. The P-type ILC scheme as well as the convergent condition for fractional-order nonlinear time-delay systems is discussed in Section 3. MATLAB/SIMULINK results are shown in Section 4. Finally, some conclusions are drawn in Section 5.

Throughout this paper, the 2-norm for the n-dimensional vector $w = (w_1, w_2, \ldots, w_n)$ is defined as $\|w\| = (\sum_{i=1}^{n} w_i^2)^{1/2}$, while the λ-norm for a function is defined as $\|\cdot\|_\lambda = \sup_{t\in[0,T]}\{e^{-\lambda t}|\cdot|\}$, where $\lambda > 0$.

2. Preliminaries

In this section, some basic definitions and lemmas are first introduced, which will be used in the following sections.

Definition 2.1. The definition of fractional integral [1,2] is described by

$$ {}_{t_0}\mathcal{D}_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-\tau)^{\alpha-1} f(\tau) d\tau, \quad \alpha > 0, $$

where $\Gamma(\cdot)$ is the well-known Gamma function.

Definition 2.2. The Caputo derivative is defined as

$$ {}_{t_0}^{C}\mathcal{D}_t^{q} f(t) = {}_{t_0}\mathcal{D}_t^{q-m} \mathcal{D}^m f(t), \quad q \in [m-1, m), $$

where $m \in \mathbb{Z}^+$, D^m is the classical m-order integral derivative.

Definition 2.3. The two-parameter function of the Mittag-Leffler [1] type is defined by

$$ E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \quad \alpha >, \ \beta > 0, \ z \in \mathbb{C}. $$

Lemma 2.4. *If the function $f(t, x)$ is continuous, then the initial value problem*

$$ \begin{cases} {}_{t_0}^{C}\mathcal{D}_t^{\alpha} x(t) = f(t, x(t)), & 0 < \alpha < 1, \\ x(t_0) = x(0) \end{cases} $$

is equivalent to the following nonlinear Volterra integral equation

$$ x(t) = x(0) + \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} f(s, x(s)) ds, $$

and its solutions are continuous [13].

Lemma 2.5. *The fractional-order differentiation or integral of the Mittag-Leffler function is*

$$ {}_{t_0}\mathcal{D}_t^{\rho}[t^{\beta-1} E_{\alpha,\beta}(\lambda t^\alpha)] = t^{\beta-\rho-1} E_{\alpha,\beta-\rho}(\lambda t^\alpha), \quad \rho < \beta. $$

3. P-type iterative learning control

Consider the following SISO fractional-order nonlinear system

$$ \begin{cases} \mathcal{D}_t^{\alpha} x_k(t) = f(x_k(t), u_k(t), t), \\ y_k(t) = g(x_k(t), u_k(t), t), \end{cases} \tag{1} $$

where $k \in \{0, 1, 2, \ldots\}$, $t \in [0, T]$, $\alpha \in (0, 1)$,

$$ 0 < \beta_1 \le g_u = \frac{\partial g(x_k(t), u_k(t), t)}{\partial u_k(t)} \le \beta_2, $$

$$ 0 < \beta_3 \le g_x = \frac{\partial g(x_k(t), u_k(t), t)}{\partial x_k(t)} \le \beta_4, $$

$$ \|f(x_k(t), u_k(t), t) - f(\bar{x}_k(t), \bar{u}_k(t), t)\| \le f_0 |u_k(t) - \bar{u}_k(t)| + f_0 \|x_k(t) - \bar{x}_k(t)\| $$

and $\beta_1, \beta_2, \beta_3, \beta_4$ and f_0 are positive constants. $x_k(t) \in \mathbb{R}^n$ is the state of the plant, and $u_k(t) \in \mathbb{R}$ and $y_k(t) \in \mathbb{R}$ are the control input and output, respectively. $\mathcal{D}_t^{\alpha}$ denotes the Caputo derivative of order α.

The design objective in this paper is to find an iterative learning control law to generate the control input $u_k(t)$ such that the system output $y_k(t)$ tracks the desired output trajectory $y_d(t)$ as accurately as possible when k goes to infinity for all $t \in [0, T]$. To this end, the P-type ILC updating law is considered.