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Data-driven optimal terminal iterative learning control

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ABSTRACT

This paper presents a data-driven optimal terminal iterative learning control (TILC) approach for linear and nonlinear discrete-time systems. The iterative learning control law is updated from only terminal output tracking error instead of entire output trajectory tracking error. The only required knowledge of a controlled system is that the Markov matrices of linear systems or the partial derivatives of nonlinear systems with respect to control inputs are bounded. Rigorous analysis and convergence proof are developed with sufficient conditions for the terminal ILC design and the results are developed for both linear and nonlinear discrete-time systems. Simulation results illustrate the applicability and effectiveness of the proposed approach.

1. Introduction

In many real plants, such as typical rapid thermal processing (RTP) systems for chemical vapor deposition (RTPCVD) [1], robot manipulators [2], and batch to batch processes [3], their main features include (1) the system executes a given task repeatedly; (2) the only available measurement is the terminal state or terminal output; (3) the ultimate control objective is also the terminal state or terminal output instead of the entire trajectory of the system output. This repeatability can be utilized to improve system control performance by iterative learning control (ILC) methods [4–7]. Conventional ILCs handle a desired output trajectory in a given time interval and they are not applicable to this control task because the exact measurement of the system state or output is impossible [1].

To overcome these problems, terminal iterative learning control (TILC) [1] is introduced to use the terminal point only at the end of every run. Now terminal ILC is becoming a new research direction of ILC with many applications [1–3,8–12]. Paper [1] presented a higher-order TILC for linear time-varying systems using basis functions to parameterize control input. However, the selection of basis function is not a trivial task for engineers in practice since there is little knowledge about the controlled system.

Recently, some optimal terminal ILC approaches (OTILC) were developed in [13,14] by introducing the explicit optimization objective into the terminal ILC design. It is noted when it is difficult to find function derivatives or if finding such derivatives are time consuming, some gradient-free optimization algorithms [15], such as the simplex method [16], quasi-gradient method [17], quasi-Newton method [18], genetic algorithm [19], differential evolution algorithm (DE) [20], and particle swarm optimization [21], can be used to solve these optimal control problems. In fact, the optimal control can be regarded as a special optimization problem, where the controlled system acts as the constraint conditions and thus it must be known exactly. As a result, a main limitation of traditional optimal control approach is the requirement of the knowledge of a perfect model. In lack of an accurate model, the monotonic convergence is no longer guaranteed. Thus, similar to the existed optimal ILC approaches [22–25], the existing optimal TILC [13,14] is lack of flexibility regarding modifications and expansions of the controlled plant. Xiong and Zhang [8] proposed a neural network model based batch-to-batch optimal TILC strategy. Selection of a proper neural network requires some efforts in practice.

This work aims to develop a new data-driven optimal terminal ILC, which is applicable to both linear and nonlinear systems. All system states are not measurable and all system matrices are unknown. We only require that the system matrices are bounded. The presented approach consists of a control input iterative learning law and a Markov matrix iterative updating law. Furthermore, the results are extended to nonlinear discrete-time systems where we only need to know the existence of the boundedness of the partial derivative of nonlinear

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system with respect to control input without requiring their exact values. The presented data-driven OTILC approach of nonlinear systems consists of a control input iterative learning law and a partial derivatives iterative updating law. The distinct features of the presented approaches are as follows: (1) The learning gain of the optimal control law is derived from the estimation of Markov matrix or partial derivatives, and can be updated iteratively using the measured I/O data only. (2) The proposed approach is considered "data-driven" or "model-free", since the controller design and analysis requires only the measurement I/O data without using any model information of the plant. Simulation study shows that the proposed approaches are effectiveness and are invariant to changes in the controlled plant.

The rest of this paper is organized as follows. Section 2 formulates the problem of optimal terminal ILC. Section 3 develops the design of the optimal TILC. Section 4 shows the stability and convergence of the TILC system with rigorous analysis. Section 5 presents the extension to nonlinear discrete-time systems are developed in Section 5. Section 6 provides simulation results to illustrate the applicability and effectiveness of the proposed approach. Finally, some conclusions are given in Section 7.

2. Problem formulation

Consider a time-varying linear discrete-time multiple input multiple output (MIMO) system as follows:

$$\begin{cases} \mathbf{x}_k(t+1) = \mathbf{A}(t)\mathbf{x}_k(t) + \mathbf{B}(t)\mathbf{u}_k(t) \\ \mathbf{y}_k(t) = \mathbf{C}(t)\mathbf{x}_k(t) \end{cases} \tag{1}$$

where $t = 0, 1, \ldots, N$ is the sampling time index, N is the finite time interval of the run-to-run system; k indicates the system repetition number; $\mathbf{x}_k(t) \in R^p$ is the system state; $\mathbf{u}_k \in R^n$ denotes the system input, which is constant at all sampling time in the same iteration; $\mathbf{y}_k(t) \in R^n$ is the system output, where only $\mathbf{y}_k(N)$ is measurable at the end of every run k. All system matrices $\mathbf{A}(t) \in R^{p\times p}$, $\mathbf{B}(t) \in R^{p\times n}$, and $\mathbf{C}(t) \in R^{n\times p}$ are unknown but bounded, that is,

$$\sup_{t=\{0,1,\ldots,N\}} ||\mathbf{A}(t)|| \le b_A, \quad \sup_{t=\{0,1,\ldots,N\}} ||\mathbf{B}(t)|| \le b_B, \quad \text{and} \quad \sup_{t=\{0,1,\ldots,N\}} ||\mathbf{C}(t)|| \le b_c.$$

Remark 1. It is noted that in system (1), the state vector $\mathbf{x}_k(t)$ is not measurable and only the terminal output quantity is measurable at the end of every iteration. Moreover, all matrices $\mathbf{A}(t)$, $\mathbf{B}(t)$, and $\mathbf{C}(t)$ are unknown, hence the usual parameter identification and state estimation methods are not applicable. So the control plant considered in this paper is different from that in [14], where all matrices $\mathbf{A}(t)$, $\mathbf{B}(t)$, and $\mathbf{C}(t)$ are assumed to be known exactly.

This linear system satisfies the following assumptions.

Assumption A1. System (1) is completely controllable.

Assumption A2. The initial state $\mathbf{x}_k(0)$ is assumed identical for every iteration k, i.e., $\forall k$, $\mathbf{x}_k(0) = \mathbf{x}_0$, which is a common assumption in traditional ILC and TILC approaches.

Thus, by solving Eq. (1), the terminal output becomes

$$\mathbf{y}_k(N) = \mathbf{C}(N)\prod_{t=0}^{N-1}\mathbf{A}(t)\mathbf{x}_k(0) + C(\mathbf{N})\sum_{i=1}^{N}\prod_{j=1}^{i-1}\mathbf{A}(N-j)\mathbf{B}(N-i)\mathbf{u}_k \tag{2}$$

By Assumption A2,

$$\Delta\mathbf{y}_k(N) = \mathbf{y}_k(N) - \mathbf{y}_{k-1}(N) = \mathbf{C}(N)\sum_{i=1}^{N}\prod_{j=1}^{i-1}\mathbf{A}(N-j)\mathbf{B}(N-i)(\mathbf{u}_k - \mathbf{u}_{k-1}) = \Xi\,\Delta u_k \tag{3}$$

where $\Xi = \mathbf{C}(N)\sum_{i=1}^{N}\prod_{j=1}^{i-1}\mathbf{A}(N-j)\mathbf{B}(N-i) \in R^{n\times n}$, and $\Delta\mathbf{u}_k = \mathbf{u}_k - \mathbf{u}_{k-1}$. Similarly, since all $\mathbf{A}(t)$, $\mathbf{B}(t)$, and $\mathbf{C}(t)$ are bounded, Ξ also is bounded. Assume that $||\Xi|| \le b_\Xi$ without loss of generality.

Remark 2. Ξ is the mapping matrix from input to output and is defined as the Markov matrix. In the following controller design, only the existence of its bound is needed.

Eq. (3) can be rewritten as

$$\mathbf{y}_k(N) = \mathbf{y}_{k-1}(N) + \Xi(\mathbf{u}_k - \mathbf{u}_{k-1}) \tag{4}$$

Remark 3. Eq. (4) is the terminal point dynamics along iteration axis of the original system (1) that has been used to construct an incremental linear mapping from system input to output in the iteration domain. Eq. (4) is a general formulation without specific dimensions of system I/O and matrices. As a result, the following presented optimal TILC based on (4) need not be re-designed in case of any change in the controlled plant.

For the following discussions, another assumption is made.

Assumption A3. The signs of all elements of Ξ are known and unchanged.

Remark 4. The sign of Ξ is similar to the control direction. Assumption A3 is a common condition in control community. For practical uncertain systems, it is easy to know the signs of all elements of Ξ by using a set of different inputs to compare the corresponding system outputs. For example, considering a SISO case for simplicity, if $\Delta u_k > 0$, the corresponding $\Delta y_k > 0$, then the sign of Ξ is positive.