PERGAMON

Nonlinear Analysis 47 (2001) 2089–2101

Nonlinear Analysis

www.elsevier.nl/locate/na

A METHOD FOR DERIVING INTEGRAL EQUATIONS USEFUL IN CONTROL CHART PERFORMANCE ANALYSIS

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Abstract

Various charting procedures have been recommended for use in monitoring one or more quality characteristics of a repetitive production process. The performance of these charts is often assessed by looking at one or more parameters of the run length distribution. For some charts an integral equation approach can be used to determine the run length parameters. A method is illustrated for deriving these integral equations which include some well known as well as some lesser known integral equations.

Key words:
Cumulative sum, control chart, CUSUM, integral equation, average run length.

1 Introduction

Performance measures of a Phase II control chart with fixed sampling intervals and fixed sample sizes are often expressed in terms of the distribution of the run length. The run length is defined as the number of the sampling stage at which the chart first signals. The most commonly used measures are the average (ARL), standard deviation ($SDRL$), and percentage points ($PPRL$) of the run length distribution. The performance measures of charts with variable sampling intervals or variable sample sizes are typically defined in terms of the parameters of the distributions of time to signal or number of observation. The three most often used methods for evaluating these performance measures are simulation, Markov chains, and integral equations. Simulation

0362-546X/01/$ - see front matter
PII: S0362-546X(01)00335-2

is a (quasi) estimation method. While it is relatively easy to implement, this method requires a relatively large amount of computer time. It should also be noted that simulation can be used in a straightforward manner to estimate some performance measures, but for others it may be quite difficult to use.

Brook and Evans (1972) demonstrated how a cumulative sum (CUSUM) charting procedure could be approximated by a Markov chain. Typically, when this approximation method appears in the literature most authors follow Brook and Evans (1972) method and choose the discrete states to be equally spaced over the in-control values of the CUSUM statistic. Champ and Rigdon (1991) suggest in their comparison of the Markov chain and integral equation methods that quadrature points could be chosen as the discrete states.

A third procedure makes use of integral equations to evaluate properties of the run length distribution of various control charting procedures. Page (1954) used this method to evaluate the run length properties of the CUSUM chart for fixed sampling intervals. Stoumbos and Reynolds (1996) and Stoumbos, Mittenthal, and Runger (2001) give integral equations for determining moments of the run length and time to signal distributions for fixed and variable sampling interval CUSUM, exponentially weighted moving average (EWMA), and sequential probability ratio test (SPRT) charts. Rigdon (1995a,b) used a similar method to derive integral equations for the multivariate EWMA chart. Little information is given in the literature for obtaining an integral equation when one is used to obtain a run length distribution parameter.

In this article, a simple and straightforward method will be demonstrated for deriving various integral equations that are used to evaluate the performance of the CUSUM chart with fixed sampling intervals. This method can be applied to several other cumulative sum type charts. The CUSUM chart is discussed in the next section. The method for deriving the integral equations used to evaluate the performance of the CUSUM charts are given in the third section. Some concluding remarks are then made.

2 Cumulative Sum Type Charts

It is assumed that information about a process is taken in the form of a continuous quality measurement, X. For illustration purposes, we will assume that X follows a normal distribution. Also, it is assumed that the quality of the process is defined in terms of the mean μ and standard deviation σ of the distribution of X. When the process is in a state of statistical control , we assume that $\mu = \mu_0$ and $\sigma = \sigma_0$. Further, if either μ_0 or σ_0 is not known, a set of m independent random samples each of size n will be available from an in-control process to estimate these parameters. Independent random samples