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## D-type anticipatory iterative learning control for a class of inhomogeneous heat equations\*

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#### ABSTRACT

In this paper, a D-type anticipatory iterative learning control (ILC) scheme is applied to the boundary control of a class of inhomogeneous heat equations, where the heat flux at one side is the control input while the temperature measurement at the other side is the control output. By transforming the inhomogeneous heat equation into its integral form and exploiting the properties of the embedded Jacobi Theta functions, the learning convergence of ILC is guaranteed through rigorous analysis, without any simplification or discretization of the 3D dynamics in the time, space as well as iteration domains. The adopted ILC scheme makes full use of the process repetition and deals with state-independent or state-dependent uncertainties. Meanwhile, due to the feedforward characteristic of ILC, the proposed scheme not only makes anticipatory compensation possible to overcome the heat conduction delay in boundary output tracking, but also eliminates the gain margin limitation encountered in feedback control. In the end, an illustrative example is presented to demonstrate the performance of the proposed ILC scheme.

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#### 1. Introduction

Iterative learning control (ILC) is an effective control technology in handling repeatable control processes. Due to its structural simplicity and model-free nature in the design process of controller, ILC has been widely used in industries, such as robotic manipulator, hard disk drives, rapid thermal processing, and chemical polymerization/crystallization (Bristow, Tharayil, & Alleyne, 2006; Chen, Moore, Yu, & Zhang, 2008; Moore, 1999; Yang & Chan, 2009). In these applications, different technologies of ILC, from the simplest P-type ILC to the much more complex higher-order PID-type ILC, have been exploited and tested in detail. ILC is playing a more and more important role in controlling repeatable processes.

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However, despite the significant progress of ILC for finitedimensional systems, studies of ILC for infinite-dimensional processes are limited and only a few related works were reported in this field. In Xu, Arastoo, and Schuster (2009), the design of P-type and D-type ILC laws for a class of infinite-dimensional linear systems is considered using semigroup theory. In Qu (2002), based on Lyapunov theory, differential-difference type ILC is augmented with proportional controller to attenuate the unknown periodic speed variation for a stretched string system on a transporter. In Zhao and Rahn (2008), the similar ILC scheme is combined with proportional-derivative controller to compensate for the unknown periodic motion on the right end for a class of axially moving material systems. While the process models are nonlinear in Qu (2002); Zhao and Rahn (2008), ILC is mainly designed for the stability maintenance of mechanical processes. Recently, under the framework of ILC, velocity boundary control of a class of quasi-linear partial differential equation (PDE) processes is considered in Huang and Xu (2011), where the convergence of output regulation is guaranteed in the steady-state stage. To the best of our knowledge, there have been no studies on the ILC design of nonlinear PDE systems for boundary-input boundaryoutput tracking tasks, which is mainly attributed to the following points: (1) the infinite-dimensional characteristic of system, (2) the interweave of 3D dynamics in the time, space, and iteration domains (Huang & Xu, 2011), and (3) the absence of universal analysis tools for ILC of distributed parameter systems.







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The heat control problem has been frequently encountered in many industrial or chemical processes, e.g., indirect heating of liquids and polymers, single-fluid batch processing, pipeline tracing, energy recovery, low pressure cogeneration, drying and heating of bulk materials, gas processing, and ebullient cooling. In the control of heat transfer equations or more general parabolic PDE systems, repetition and correction mechanisms are as common as in lumped parameter systems modeled by ordinary differential equations (ODEs). Examples include the batch heat treatment furnace (Tiwari, Mukhopadhyay, & Sanyal, 2005), tubular heat exchangers (Alvarez, Yebra, & Berenguel, 2007), batch thermal sterilization processes (Akterian, 1999), temperature control of tokamak plasmas (Xu et al., 2009), and many other batch processes. In Boskovic, Krstic, and Liu (2001), Fridman and Orlov (2009), Jiang, Nguyen, and Prudhomme (2005), Krstic and Smyshlyaev (2008), Pisano and Orlov (2012), the boundary control problem for heat processes was studied under strict assumptions on the admitted uncertainties and perturbations. Moreover, the most important characteristic of processes, i.e., the repetitiveness, will be ignored, when the control schemes proposed in Boskovic et al. (2001), Fridman and Orlov (2009), Jiang et al. (2005), Krstic and Smyshlyaev (2008), Pisano and Orlov (2012) are applied for batch processes.

In this paper, a D-type anticipatory ILC scheme is applied to the boundary control of a class of inhomogeneous heat equations, where the nonlinear heat source is state-independent or statedependent. Under a repeatable process environment, the heat flux at one side is considered as the control input while the temperature measurement at the other side is considered as the control output. First, the heat conduction equation is transformed into its integral form, based on which the input-output error dynamics are presented clearly. Then, rigorous analysis is performed to exploit the properties of the embedded Jacobi Theta functions in the error dynamics. With practical assumption on the uncertainties of heat equations, these properties facilitate the consequent ILC design and convergence analysis. As a result, we can iteratively tune the heat flux boundary condition on one side such that the boundary output at the other side can track the desired reference pointwisely.

It is worth noticing that we neither simplify the infinitedimensional heat equations to finite-dimensional ODE systems as in Huang and Xu (2011) nor replace them by the discrete-time equivalences as in Cichy, Galkowski, Rogers, and Kummert (2011). On the one hand, in Huang and Xu (2011), the infinite-dimensional heat equation is simplified as a finite-dimensional ODE system at steady-state stage, which is only applicable for the set-point control task. If tracking control is considered as in our paper, model simplification has the disadvantage of not taking relevant heat conduction issues into account, namely, the distributed parameter characteristic of heat conduction process is neglected, thus is ineffective here. On the other hand, in Cichy et al. (2011), explicit discretization is conducted for a class of linear heat equations to derive a multidimensional discrete linear system, based on which the ILC law is designed. The models obtained by the approach are of the local type and hence the state-space dimension is low and finite. It is obviously necessary to ensure that they adequately capture the dynamics of the defining PDEs. Since this problem has not been addressed in Cichy et al. (2011), there is much further research to be done on this approach to ensure that an adequate discrete model for design is produced in the most efficient way. Meanwhile, numerical instability must be prevented by imposing limits on the time and space discretization periods. Although it can be calculated by means of some numerical analysis methods or software tools, it also hinders us to apply the proposed ILC scheme conveniently, which might be a disadvantage of ILC with model discretization. In particular, when the heat equation is nonlinear and/or possesses some structural uncertainties, the analysis method proposed for linear systems in Cichy et al. (2011) will lose its efficacy. In our work, the ILC design and analysis is performed for the original heat conduction equation, thus a class of "real" distributed parameter systems. Without checking the numerical stability or the adequate approximation property of the reduced plant, the proposed control scheme is applicable directly for the boundary tracking control of nonlinear heat equations.

Moreover, owing to the fact that ILC is a feedforward control, the proposed scheme not only makes anticipatory compensation possible to overcome the heat conduction delay in boundary output tracking, but also eliminates the gain margin limitation encountered in feedback control.

Throughout the paper, denote **R** the set of real numbers,  $\mathcal{N}$  the set of nonnegative integers, Q the set of  $\{(x, t)|0 < x < 1, 0 < t \leq T\}$ ,  $\overline{Q}$  the closed set of Q, namely,  $\{(x, t)|0 \leq x \leq 1, 0 \leq t \leq T\}$ ,  $\overline{C}^n([l_1, l_2], \mathbf{R})$  the set of scalar continuous functions as n = 0 or continuously differentiable functions as n = 1 in the interval  $[l_1, l_2]$ , and  $\mathcal{H}(E, \mathbf{R})$  an infinite-dimensional Hilbert space of scalar functions defined on a domain *E*. For the function  $v(x, t) \in \mathcal{H}(\overline{Q}, \mathbf{R})$ ,  $v_z$  denotes its partial derivative with respect to variable *z*, e.g.,  $v_t = \frac{\partial v}{\partial t}$  and  $v_{xx} = \frac{\partial^2 v}{\partial x^2}$ . For simplicity, we sometimes use the abbreviation *v* instead of v(x, t) below. For a time-related function  $f(t) \in \mathbf{R}$ , |f(t)| takes its absolute value, and  $|f|_{\lambda} \triangleq \sup_{\tau \in [0,T]} e^{-\lambda \tau} |f(\tau)|$  denotes its  $\lambda$ -norm, where  $\lambda$  is a positive constant.

#### 2. System description and problem statement

Consider the heat flux boundary control of the following onedimensional inhomogeneous heat equation under a repeatable environment (Cannon, 1984)

$$\begin{cases} v_t^i(x,t) = v_{xx}^i(x,t) + F(x,t,v^i(x,t),v_x^i(x,t)), & (x,t) \in Q, \\ v^i(x,0) = f(x), & x \in (0,1), \\ v_x^i(0,t) = u^i(t), & t \in [0,T], \\ v_x^i(1,t) = g(t), & t \in [0,T], \end{cases}$$
(1)

where  $t \in [0, T]$  is the time,  $x \in [0, 1]$  is the spatial coordinate,  $v^i(x, t) \in \mathscr{H}(\overline{Q}, \mathbb{R})$  is the temperature measurement at the time t and the position x, and  $i \in \mathscr{N}$  is the iteration number. Moreover,  $u^i \in C^0([0, T], \mathbb{R}), g \in C^0([0, T], \mathbb{R})$ , and  $f \in C^1((0, 1), \mathbb{R})$ such that f and  $f_x$  are bounded. The unknown function  $F(x, t, v^i, v_x^i)$ is defined on the set  $\Theta = \{(x, t, v^i, v_x^i) \mid (x, t) \in \overline{Q}, -\infty < v^i, v_x^i < \infty\}$ . Assuming the finiteness of  $|v^i|$  and  $|v_x^i|$ , the function  $F(x, t, v^i, v_x^i)$  is uniformly Hölder continuous<sup>2</sup> in x and t for each compact subset of Q. In addition, there exists an unknown constant  $C_F$  such that

$$|F(x, t, p_1, q_1) - F(x, t, p_2, q_2)| \le C_F\{|p_1 - p_2| + |q_1 - q_2|\}, \quad (x, t) \in \overline{Q}$$
(2)

holds for all  $(p_i, q_i)$ , i = 1, 2, namely,  $F(x, t, v^i, v^i_x)$  is Lipschitz continuous in the state-dependent variables  $v^i$  and  $v^i_x$ . It is easy to write down examples of such functions, for example,  $F(x, t, p, q) = \sin(xt)\cos(p) + \cos(xt)\sin(q)$ . In the context of heat conduction or diffusion, the uncertainty function F can be interpreted as a heat source or sink. For most applications, the nonlinear

<sup>&</sup>lt;sup>2</sup> A real or complex-valued function  $\chi$  on *d*-dimensional Euclidean space is Hölder continuous when there are nonnegative real constants *C* and  $\alpha$  such that  $|\chi(x) - \chi(y)| \le C|x - y|^{\alpha}$  for all *x* and *y* in the domain of  $\chi$  (Evans, 1998). The number  $\alpha$  is called the exponent of the Hölder condition. If  $\alpha = 1$ , then the function satisfies a Lipschitz condition. If  $\alpha = 0$ , then the function is bounded.

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