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Performance analysis of the adaptive line enhancer with multiple sinusoids in noisy environment

R.L. Campbell Jr.[a], N.H. Younan[b,*], J. Gu[b,1]

[a]Information Technology Laboratory, U.S. Army Corps of Engineers, Vicksburg, MS 39180, USA
[b]Department of Electrical and Computer Engineering, Mississippi State University, Box 9571, Mississippi State, MS 39762-9571, USA

Abstract

In this paper, the performance analysis of the adaptive line enhancer when the input signal consists of multiple sinusoids embedded in noise is investigated. The performance is evaluated in terms of the signal-to-noise ratio gain at the filter's output. It is shown that, for multiple sinusoids, this gain is not only a function of the filter length, but also of three additional factors — the number of sinusoids, the noise power, and the amplitude of each sinusoid. Simulation results for a dual noisy sinusoidal input are presented to illustrate the validity of this analysis.

1. Introduction

In general, the physical telecommunications infrastructure in the United States is primarily copper between local telecommunication companies and their users. Although the late-90's ideologues forecasted a timely transition from old copper telephone infrastructure to fiber, where there was talk of fiber-to-the-curb or fiber to each person's home, it appears now that there is little to no push by the local telephone exchanges to change the distribution medium to their customers. It is copper and will remain copper for sometime to come. This places the local exchange in a precarious situation in terms of competition with its natural nemesis; the cable television provider [1]. Aside from power cables, telephone wires and coaxial cable television are commonly distributed to the majority of homes. However, a coaxial cable has a distinct advantage over a two-wire telephone connection, since a coaxial cable is naturally shielded [8,9]. Therefore, the standard telephone connection faces greater noise and will pose a particular design problem to those telephone engineers that are working to bring Internet access to densely populated areas [1].

In competition with this service, local telephone companies provide a service known as Asynchronous Digital Subscriber Line (ADSL) over their non-coaxial infrastructure [7]. ADSL has been touted as the solution for an end-user that plans to use telephone wires to gain access, since it provides a high speed uplink and a much higher speed downlink, which is ideal for web browsing [1]. Since ADSL occupies the 0–500 kHz band, a broadband noise management approach is in order [8]. To provide this service reliably, they must combat a number of problems including line attenuation, group delay, and noise, with the noise being the unique obstacle.

An interesting application for performing noise reduction on a telephone wire connection is the use of

* Corresponding author. Tel.: +1-601-325-3912; fax: +1-601-325-2298.
E-mail address: younan@ece.msstate.edu (N.H. Younan).
[1] J. Gu is with Ansoft Corporation, 669 River Drive, Suite 200, Elmwood park, NJ 07407, USA.

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adaptive filtering, where the adaptive line enhancer (ALE) is chosen in this investigation due to its simplicity and ease of implementation [11]. Although the ALE is known to be applicable to narrowband signals in broadband noise [3,5], it is not clear as to why the ALE does not extend well to a problem like this one, since the ALE provides enhancement on a sinusoid by sinusoid basis. In other terms, it is not evident as to why its performance does not extrapolate well to a host of sinusoids (i.e. a band).

Adaptive line enhancement techniques for tracking sinusoids in noise have been widely used and their performance for improving the signal-to-noise ratio (SNR) has been also studied [4,10,2,12]. The main goal of this paper is to explore the limitations of the adaptive line enhancer by analytically describing its performance to predict the algorithm's SNR gain. For multiple sinusoids, it is shown that this gain is not only a function of the filter length L, but also of three other additional factors — the number of sinusoids, the noise power, and the amplitude of each sinusoid. Furthermore, it is shown that the gain can be exactly predicted for all L corresponding to any integer multiple of the noise-free signal's fundamental frequency. At other values of L, the gain is approximated. Simulation results for a noisy sinusoidal input are presented to illustrate the validity of this analysis. Note that the model of a fundamental and its harmonics are used for signaling in this simulation. Furthermore, since a set of harmonic sinusoids is a subset of the entire domain of individual sinusoids, the extrapolation of individual sinusoids into a broadband is somehow similar to the extrapolation of groups of harmonic sinusoids into a broadband.

2. Analysis

2.1. General concept

The ALE is one of the most useful applications of adaptive filtering. It is a method of optimal filtering that can be applied to enhance noise corrupted signals. It is designed to suppress the broadband noise component of an input signal, while passing the narrowband component, such as a sinusoid, with little distortion [11]. It is also capable of automatically turning itself off when no SNR improvement is achieved. In contrast to other techniques, the ALE does not require a priori knowledge of the signal or noise. The block diagram of the ALE is shown in Fig. 1. Assuming that the input signal $d(n)$ consists of a deterministic or noise-free component plus an additive noise component, i.e., a narrowband signal plus broadband noise, and denote $x(n)$ as a delayed version of $d(n)$, then $x(n)$ decorrolates the broadband noise while leaving the narrowband component correlated. Ideally, the output of the adaptive filter, $y(n)$, is an estimate of the noise-free input signal. The computational algorithm for the ALE is [6]

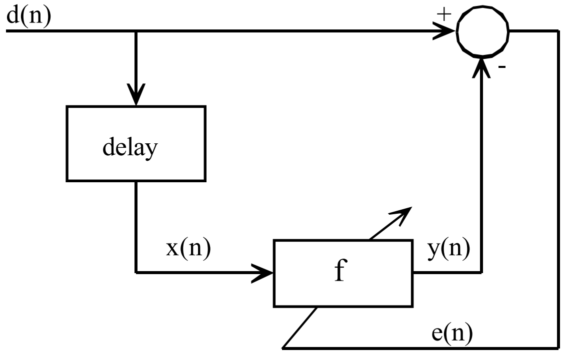

Fig. 1. Model of the adaptive line enhancer.

$$
\begin{aligned}
y(n) &= \sum_{i=0}^{L-1} f_i(n)x(n-i), \quad x(n) = d(n-\Delta), \\
f_i(n+1) &= f_i(n) + 2\alpha e(n)x(n-i), \\
e(n) &= d(n) - y(n),
\end{aligned} \tag{1}
$$

where α is the adaptation parameter that controls the speed of convergence and the stability of the filter, Δ is the decorrelation parameter which depends on the correlation lag of the input signal components, L is the filter length, $f_i(n)$ is the ith set of filter coefficients, and $n \in [0, N-1]$ with N being the data length. Note that the adaptation parameter α plays a major role and its value results in a compromise or a tradeoff between stability and convergence, thus eventually affecting the steady-state error.

2.2. SNR derivation

It is of interest to explore the limitations of the ALE algorithm, described by Eq. (1), for multiple