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A multiprocess performance analysis chart based on the incapability index C_{pp}: an application to the chip resistors

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Abstract

Statistical process control charts, such as the $\bar{X}$, R, S^2, S, and MR charts, have been widely used in the manufacturing industry for controlling/monitoring process performance, which are essential tools for any quality improvement activities. Those charts are easy to understand, which effectively communicate critical process information without using words and formula. In this paper, we introduce a new control chart, called the C_{pp} multiple process performance analysis chart (MPPAC), using the incapability index C_{pp}. The C_{pp} MPPAC displays multiple processes with the departure, and process variability relative to the specification tolerances, on one single chart. We demonstrate the use of the C_{pp} MPPAC by presenting a case study on some resistor component manufacturing processes, to evaluate the factory performance.

1. Introduction

Process capability indices (PCIs) have been widely used in various manufacturing industries, to provide numerical measures on process potential and process performance. The two most commonly used process capability indices are C_p and C_{pk} introduced by Kane [1]. These two indices are defined in the following:

$$C_p = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sigma},$$

$$C_{pk} = \min\left\{\frac{\mathrm{USL} - \mu}{3\sigma}, \frac{\mathrm{LSL} - \mu}{3\sigma}\right\},$$

where USL and LSL are the upper and the lower specification limits, respectively, μ is the process mean, and σ is the process standard deviation. The index C_p measures the process variation relative to the production tolerance, which reflects only the process potential. The index C_{pk} measures process performance based on the process yield (percentage of conforming items) without considering the process loss (a new criteria for process quality championed by Hsiang and Taguchi [2]). Taking into the consideration of the process departure (which reflects the process loss), Chan et al. [3] developed the index C_{pm}, which measures the ability of the process to cluster around the target. The index C_{pm} is defined as:

$$C_{pm} = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}},$$

where T is the target value, and $d = (\mathrm{USL} - \mathrm{LSL})/2$ is half of the length of the specification interval (LSL, USL).

Based on the index C_{pm}, Greenwich and Jahr-Schaffrath [4] introduced an incapability index, called C_{pp}, which is a simple transformation of the Taguchi index C_{pm}. The index C_{pp} is defined as:

$$C_{pp} = \left(\frac{1}{C_{pm}}\right)^2 = \left(\frac{\mu - T}{D}\right)^2 + \left(\frac{\sigma}{D}\right)^2,$$

where $D = d/3$. Some commonly used values of C_{pp}, 9.00 (process is incapable), 4.00 (process is incapable), 1.00 (process is normally called capable), 0.57 (process is normally called satisfactory), 0.44 (process is normally called good), and 0.25 (process is normally called super), and the corresponding C_{pm} values are listed in Table 1.

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Table 1
Some commonly used C_{pp} and equivalent C_{pm}

C_{pp}	C_{pm}
9.00	0.33
4.00	0.50
1.00	1.00
0.57	1.33
0.44	1.50
0.25	2.00

Table 2
Some commonly used precision requirements

Quality condition	Precision requirement
Capable	$0.56 \leqslant C_{ip} \leqslant 1.00$
Satisfactory	$0.44 \leqslant C_{ip} \leqslant 0.56$
Good	$0.36 \leqslant C_{ip} \leqslant 0.44$
Excellent	$0.25 \leqslant C_{ip} \leqslant 0.36$
Super	$C_{ip} \leqslant 0.25$

If we denote the first term $(\mu - T)^2/D^2$ as C_{ia}, and the second item σ^2/D^2 as C_{ip}, then C_{pp} can be rewritten as $C_{pp} = C_{ip} + C_{ia}$. The sub-index C_{ip} measures the relative variability, which has been referred to as the imprecision index. Some commonly used values of C_{ip}, 1.00, 0.56, 0.44, 0.36, and 0.25, and the corresponding quality conditions are listed in Table 2. Note that those values of C_{ip} are equivalent to $C_p = 1.00$, 1.33, 1.50, 1.67, and 2.00 respectively, covering a wide range of the precision requirements used for most real-world applications.

On the other hand, the sub-index C_{ia} measures the relative departure, which has been referred to as the inaccuracy index. The advantage of using the index C_{pp}, is that it provides an uncontaminated separation between information concerning the process precision and process accuracy. The separation suggests a direction the practitioners may consider on the process parameters to improve the process quality.

Based on the sub-indices C_{ip} and C_{ia}, we introduce a control chart called the C_{pp} multiple process performance analysis chart (MPPAC), using the incapability index C_{pp}. The C_{pp} MPPAC displays multiple processes with the relative departure, and process variability relative to their specification tolerances on one single chart. We demonstrate the use of the C_{pp} MPPAC by presenting a case study taken from a resistor component manufacturing company located on an Industrial Park in Taiwan, to evaluate the factory performance.

2. Estimation of C_{ip}, C_{ia}, C_{pp}

2.1. Estimation of C_{ip}

To estimate the process imprecision, we consider the natural estimator $\hat{C}_{ip}$ defined in the following, where the sample standard deviation S_{n-1} is calculated as $S_{n-1} = \left[\sum (X_i - \bar{X})^2/(n-1)\right]^{1/2}$, which is the conventional estimator of the process standard deviation σ,

$$\hat{C}_{ip} = \frac{1}{n-1}\sum_{i=1}^{n}\frac{\left(X_i - \bar{X}\right)^2}{D^2} = \frac{S_{n-1}^2}{D^2}.$$

The natural estimator $\hat{C}_{ip}$ can be rewritten as:

$$\hat{C}_{ip} = \frac{C_{ip}}{n-1}\frac{(n-1)\hat{C}_{ip}}{C_{ip}} = \frac{C_{ip}}{n-1}\sum_{i=1}^{n}\frac{\left(X_i - \bar{X}\right)^2}{\sigma^2}.$$

If the process characteristic is normally distributed, Pearn and Lin [5] showed the natural estimator $\hat{C}_{ip}$ distributed as $[C_{ip}/(n-1)]\chi^2_{n-1}$, where χ^2_{n-1} is a chi-square distribution with $(n-1)$ degrees of freedom. Pearn and Lin [5] showed that the natural estimator $\hat{C}_{ip}$ is the uniformly minimum-variance unbiased estimate (UMVUE) of C_{ip}, which is consistent, and asymptotically efficient. Pearn and Lin [5] also showed that the statistic $\sqrt{n}(\hat{C}_{ip} - C_{ip})$ converges to $N(0, 2C_{ip}^2)$ in distribution. Thus, in real-world applications, using $\hat{C}_{ip}$ which has all desired statistical properties as an estimate of C_{ip}, would be reasonable.

Note that by multiplying the constant $c_n = (n-1)/n$ to the UMVUE $\hat{C}_{ip}$, we can obtain the maximum likelihood estimate (MLE) of C_{ip}. Pearn and Lin [5] showed that the MLE $\hat{C}'_{ip}$ is consistent, asymptotically unbiased and efficient. They also showed that the statistic $\sqrt{n}(\hat{C}'_{ip} - C_{ip})$ converges to $N(0, 2C_{ip}^2)$ in distribution.

Since the constant $c_n < 1$, then the MLE $\hat{C}'_{ip} = c_n\hat{C}_{ip}$ underestimates C_{ip} but with smaller variance. In fact, we may calculate the mean square error MSE $(\hat{C}'_{ip}) = [(2n-1)/n^2](C_{ip})^2$. Hence, MSE $(\hat{C}_{ip}) - \text{MSE}(\hat{C}'_{ip}) = [(3n-1)/n^2(n-1)](C_{ip})^2 > 0$, for all sample size n. Therefore, the MLE $\hat{C}'_{ip}$ has a smaller mean square error than that of the UMVUE $\hat{C}_{ip}$, hence is more reliable, particularly, for short production run applications (such as accepting a supplier providing short production runs in QS-9000 certification). For short run applications (with $n \leqslant 35$) we recommend using the MLE $\hat{C}'_{ip}$ rather than the UMVUE $\hat{C}_{ip}$. For other applications with sample sizes $n > 35$, the difference between the two estimators is negligible (less than 0.52%).

2.2. Estimation of C_{ia}

For the process inaccuracy index C_{ia}, we consider the natural estimator $\hat{C}_{ia}$ defined as the following

$$\hat{C}_{ia} = \frac{(\bar{X} - T)^2}{D^2},$$

where the sample mean $\bar{X} = \sum_{i=1}^{n} X_i/n$ is the conventional estimator of the process mean μ. We note that the estimator $\hat{C}_{ia}$ can be written as the following: