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Performance analysis and optimization of straight taper fins with variable heat transfer coefficient

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Abstract

In the present paper, the thermal analysis and optimization of straight taper fins has been addressed. With the help of the Frobenius expanding series the temperature profiles of longitudinal fin, spine and annular fin have been determined analytically through a unified approach. Simplifying assumptions like length of arc idealization and insulated fin tip condition have been relaxed and a linear variation of the convective heat transfer coefficient along the fin surface has been taken into account. The thermal performance of all the three types of fin has been studied over a wide range of thermo-geometric parameters. It has been observed that the variable heat transfer coefficient has a strong influence over the fin efficiency. Finally, a generalized methodology has been pointed out for the optimum design of straight taper fins. A graphical representation of optimal fin parameters as a function of heat duty has also been provided.

1. Introduction

Fins or extended surfaces are widely used to augment the rate of heat transfer from the primary surface to the ambient medium in a large variety of thermal equipment. An accurate analysis of heat transfer in fins has become crucial with the growing demand of high performance of heat transfer surfaces with progressively smaller weights, volumes, initial and running cost of the system. Over the years different fin shapes have been evolved depending upon the application and the geometry of the primary surface. Kern and Kraus [1] have identified three main fin geometries. These are longitudinal fins, radial or circumferential fins and pin fins or spines. For any of the above geometry, fins with straight profile or constant thickness are a common choice as they can be manufactured easily. The thermal design of a constant thickness fin is also relatively simple. However, in any fin the temperature difference reduces from the fin base to fin tip. Accordingly, a saving of fin material can be obtained by progressively narrowing down the fin section. This has initiated a lot of exercises for the determination of optimum fin shapes so that the fin volume is minimum for a given rate of heat dissipation or the rate of heat dissipation is maximum for a given fin volume. The criteria for optimum fin profile under convective conditions was first proposed by Schmidt [2] based on a physical reasoning. Later on Duffin [3] proved Schmidt criteria using calculus of variation. Both Schmidt [2] and Duffin [3] estimated the fin surface area neglecting the profile curvature. This has formed a major assumption in further exercises of fin optimization and is known as length of arc idealization (LAI) in literature. LAI was used for optimizing fin shapes under convecting, radiating, convective-radiating condition [4], for fins with heat generation [5] and for variable thermal conductivity.

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Nomenclature

Bi	Biot number, $h_{av}y_b/k$
h	heat transfer coefficient (W/m^2 K)
h_b	heat transfer coefficient at the fin base (W/m^2 K)
h_{av}	average heat transfer coefficient (W/m^2 K)
h_e	heat transfer coefficient at the tip (W/m^2 K)
J	Jacobian
k	thermal conductivity of fin material (W/m K)
L	length of the fin, see Fig. 1 (m)
m, n	constants used in Eq. (1)
m_p	dimensionless parameter defined in Eq. (4)
q	actual heat transfer rate of a fin (W)
Q	non-dimensional actual heat transfer rate defined in Eq. (20)
q_i	ideal heat transfer rate (W)
Q_i	non-dimensional ideal heat transfer rate defined in Eq. (23)
r_i	base radius, see Fig. 1c (m)
R_i^*	dimensionless radius parameters, $h_{av}r_i/k$
r_o	outer radius of an annular fin (m)
R_o	radius ratio, r_o/r_i
T	local fin temperature (K)
T_a	temperature of the surrounding gas medium of the fins (K)
T_t	tip temperature (K)
T_w	fin base temperature (K)
U	dimensionless fin volume defined in Eq. (25)
V	fin volume (m^3)
x, y	Cartesian coordinates (m)
X, Y	$x/L, y/y_b$ respectively
y_b	fin base semi-thickness (m)
y_e	fin tip semi-thickness (m)
Z_0	fin parameter, $\sqrt{Bi}/\psi$

Greek symbols

α	parameter defined in Eq. (4)
β	tip loss parameter, $2\psi/(1+\varepsilon)$
δ	dimensionless parameter, Bi/ψ
ε	ratio of base to tip heat transfer coefficient, h_b/h_e
η	fin efficiency
λ	ratio of tip to base fin thickness, y_e/y_b
ν	parameter defined in Eq. (4)
ψ	aspect ratio, y_b/L
θ	dimensionless temperature, $(T-T_a)/(T_w-T_a)$
θ_0	excess base temperature, T_w-T_a (K)
θ_t	dimensionless tip temperature, $(T_t-T_a)/(T_w-T_a)$
ξ	dimensionless parameter defined in Eq. (4)

Maday [6] in his pioneering analysis proposed the correct formulation for the optimization of longitudinal fin with the elimination of LAI and obtained a profile much different from Duffin [3]. Guceri and Maday [7] further extended this analysis for radial fins.

However, fin shapes determined by the above procedure are complex and difficult to manufacture. These fins have structurally weak slender tips, which do not substantially contribute to the overall heat dissipation. This has resulted in a parallel effort to design optimum fins where the fin shape is specified a priori and fin dimensions are determined to give maximum heat dissipation for a given fin volume.