Control Engineering Practice 21 (2013) 1310–1320

Contents lists available at SciVerse ScienceDirect

Control Engineering Practice

journal homepage: www.elsevier.com/locate/conengprac

Robust finite frequency range iterative learning control design and experimental verification ☆

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ARTICLE INFO

Article history:
Received 12 September 2012
Accepted 23 May 2013
Available online 5 July 2013

Keywords:
Iterative learning control
Controller design
Kalman–Yakubovich–Popov lemma
Linear matrix inequalities
Robust control

ABSTRACT

Iterative learning control is an application for two-dimensional control systems analysis where it is possible to simultaneously address error convergence and transient response specifications but there is a requirement to enforce frequency attenuation of the error between the output and reference over the complete spectrum. In common with other control algorithm design methods, this can be a very difficult specification to meet but often the control of physical/industrial systems is only required over a finite frequency range. This paper uses the generalized Kalman–Yakubovich–Popov lemma to develop a two-dimensional systems based iterative learning control law design algorithm where frequency attenuation is only imposed over a finite frequency range to be determined from knowledge of the application and its operation. An extension to robust control law design in the presence of norm-bounded uncertainty is also given and its applicability relative to alternative settings for design discussed. The resulting designs are experimentally tested on a gantry robot used for the same purpose with other iterative learning control algorithms.

1. Introduction

Many industrial systems execute a task over a finite duration, reset to the starting location and then completes this task over and over again. Each execution is known as a trial or pass and the duration the trial length. Once a trial has been completed all data generated is available to update the control signal for the next trial and thereby improve performance from trial-to-trial. This area is known as Iterative Learning Control (ILC) and since the initial work, widely credited to Arimoto, Kawamura, and Miyazaki (1984), has been an established area of control systems research and application, where one starting point for the literature is the survey papers (Ahn, Chen, & Moore, 2007; Bristow, Tharayil, & Alleyne, 2006). Major application areas include robotics, with recent work in, for example (Barton & Alleyene, 2011), flexible valve actuation for non-throttled engine load control (Heinzen, Gillella, & Sun, 2011) and also a transfer from engineering to next generation healthcare for robotic-assisted upper limb stroke rehabilitation with supporting clinical trials (Freeman et al., 2009, Freeman, Rogers, Hughes, Burridge, & Meadmore, 2012).

A significant part of the currently published ILC research starts from a linear time-invariant discrete model of the system dynamics in either state-space or shift operator/transfer-function form. In this case, one way to do control law design is, since the trial duration is finite, to define super-vectors for the variables. For example, let $y_k(p)$ be the scalar, for ease of presentation with a natural extension to the vector case, output on trial k, which is of length $\alpha<\infty$. Then the super-vector, for linear dynamics and systems with a nonzero first Markov parameter, is

$$Y_k=[y_k(0)\ \ y_k(1)\ \ \cdots\ \ y_k(\alpha-1)]^\top,$$

and the ILC problem can hence be written as a system of linear difference equations with updating in k.

The current trial error is the difference between the supplied reference signal and the trial output and, since the trial length is finite, trial-to-trial error convergence is independent of the state matrix. This, in turn, could lead to unacceptable along the trial dynamics. In the lifting approach, this problem can be addressed by applying a feedback control law to stabilize the system and/or improve transient performance and then design the ILC law for the controlled system. An alternative is to use a two-dimensional (2D) systems setting where ILC can be represented in this form with one direction of information propagation from trial-to-trial and the other along the trial. Given that the trial length is finite, ILC fits naturally into the repetitive process setting for analysis, where these processes (Rogers, Gałkowski, & Owens, 2007) have their origins in the mining and metal rolling industries and a substantial body of systems theory and control law design algorithms exists for them.

☆This work is partially supported by Ministry of Science and Higher Education in Poland, Grant no. N N514 636540.

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http://dx.doi.org/10.1016/j.conengprac.2013.05.011

Using the repetitive process setting, it is possible to simultaneously design a control law for trial-to-trial error convergence and along the trial performance. The method is to use a form of stability for repetitive processes that demands a bounded-input bounded-output property independent of the trial length. Control laws designed in this setting have been experimentally tested on a gantry robot replicating a robotic pick and place operation that often arises in industrial applications to which ILC is applicable (Hładowski et al., 2010, 2012).

This previous ILC design in a repetitive process setting includes a stability condition that requires frequency gain attenuation over the complete frequency range and hence, by analogy with the standard linear systems case, is a very strict condition, especially as the reference signal many only have significant frequency content over a finite range. This paper develops a new control law design method where frequency attenuation is enforced over a finite range to be decided by frequency decomposition of the reference signal. The design makes use of the generalized Kalman–Yakubovich–Popov (KYP) lemma to establish the equivalence between frequency domain inequalities over finite and/or semi-finite frequency ranges for a transfer-function and a linear matrix inequality (LMI) defined in terms of its state-space realization (Iwasaki & Hara, 2005). The resulting design algorithm is experimentally applied to the gantry robot used in Hładowski et al. (2010, 2012), including the extension to robust design using a norm bounded uncertainty representation that offers advances not possible for the same problem in the lifted setting.

The following notation is used throughout this paper. For a matrix X, $X^\top$ and X^* denote its transpose and complex conjugate transpose respectively. The null and identity matrices with appropriate dimensions are denoted by 0 and I respectively. Moreover, the notation $X \succcurlyeq Y$ (respectively $X \succ Y$) means that the matrix $X-Y$ is positive semi-definite (respectively, positive definite). Also sym$\{X\}$ is used to denote the symmetric matrix $X+X^\top$ and $X^\perp$ denotes the orthogonal complement of the matrix X, that is, a matrix whose columns form a basis of the nullspace of X. The symbol $(\star)$ denotes block entries in symmetric matrices and $\rho(\cdot)$ denotes the spectral radius of its matrix argument, that is, if h_i, $1 \le i \le h$, is an eigenvalue of the $h \times h$ matrix H then $\rho(H) = \max_{1 \le i \le h} |\lambda_i|$.

2. Preliminaries

The systems considered are linear and time-invariant and, after sampling if necessary, described in the ILC setting by the nominal state-space model

$$x_k(p+1) = Ax_k(p) + Bu_k(p),$$
$$y_k(p) = Cx_k(p), \ 0 \le p \le \alpha-1, \tag{1}$$

where $k \ge 0$ is the trial number, $\alpha < \infty$ is the number of samples along the trial, $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the output vector and $u_k(p) \in \mathbb{R}^l$ is the control input vector. The uncertainty associated with the dynamics is modeled as additive perturbations ΔA, ΔB and ΔC to the matrices A, B and C, respectively, giving the model for design as

$$x_k(p+1) = (A+\Delta A)x_k(p) + (B+\Delta B)u_k(p),$$
$$y_k(p) = (C+\Delta C)x_k(p). \tag{2}$$

These perturbations are assumed to be of the norm-bounded form, that is,

$$\Delta A = H_1 \mathcal{F}(p) E_1, \ \Delta B = H_1 \mathcal{F}(p) E_2, \ \Delta C = H_2 \mathcal{F}(p) E_1, \tag{3}$$

where H_1, H_2, E_1 and E_2 are known real constant matrices of compatible dimensions and $\mathcal{F}(p)$ is an uncertain perturbation satisfying

$$\mathcal{F}(p)\mathcal{F}^\top(p) \preccurlyeq I. \tag{4}$$

Also $\mathcal{F}(p)$ is assumed to be independent of the trial number k and hence the allowable uncertainties can vary along each trial but are assumed constant from trial-to-trial.

Let $r(p) \in \mathbb{R}^m$ denote the reference vector and hence the error on trial k is

$$e_k(p) = r(p) - y_k(p). \tag{5}$$

A commonly used ILC strategy to construct the current trial input is as the sum of that used on the previous one plus a corrective term, that is,

$$u_{k+1}(p) = u_k(p) + \Delta u_k(p), \ k \ge 0, \tag{6}$$

where $\Delta u_k(p)$ denotes the modification term to the control input used on the previous trial.

In the lifting approach (the survey papers Ahn et al., 2007; Bristow et al., 2006 are one starting point for the literature) the next step in design is to define the super-vector corresponding to yk of the previous section as

$$E_k = [e_k^\top(0) \ e_k^\top(1) \ \cdots \ e_k^\top(\alpha-1)]^\top,$$

and proceed to write the controlled dynamics in the form $E_{k+1} = \mathcal{Q}E_k$, where $\mathcal{Q}$ is a block lower triangular matrix whose non-zero entries are formed from the Markov parameters of the system state-space model. This approach subsumes the along the trial dynamics and assumes that any requirements beyond trial-to-trial error convergence arising in a particular application are, if required, met by first designing a feedback control loop for the system and then applying lifting to the resulting state-space model. This paper uses the repetitive process setting for ILC design where it is possible to simultaneously design a control law for trial-to-trial error convergence and performance along the trials and the extension to robust design is straightforward. A brief discussion of the advantages of robust ILC control law design in this setting as opposed to lifting designs is also given.

To obtain a repetitive process description of the ILC dynamics introduce, for analysis purposes only, the vector

$$\eta_{k+1}(p+1) = x_{k+1}(p) - x_k(p).$$

Suppose also that in the ILC law (6)

$$\Delta u_{k+1}(p) = K_1 \eta_{k+1}(p+1) + K_2 e_k(p+1), \tag{7}$$

where K_1 and K_2 are matrices to be designed. Application of this control law to (2) gives the controlled dynamics state-space model

$$\eta_{k+1}(p+1) = \widehat{A}\eta_{k+1}(p) + \widehat{B}_0 e_k(p),$$
$$e_{k+1}(p) = \widehat{C}\eta_{k+1}(p) + \widehat{D}_0 e_k(p), \tag{8}$$

where

$$\widehat{A} = (A+\Delta A) + (B+\Delta B)K_1, \quad \widehat{D}_0 = (I-(C+\Delta C) \times (B+\Delta B)K_2),$$
$$\widehat{B}_0 = (B+\Delta B)K_2, \quad \widehat{C} = -(C+\Delta C) \times ((A+\Delta A) + (B+\Delta B)K_1). \tag{9}$$

The state-space model (8) is that of a discrete linear repetitive process with pass output and state vectors e_{k+1} and η_{k+1}, respectively, once the initial conditions are specified, that is, the pass state initial vector $\eta_k(0)$, $k \ge 1$, and the initial pass profile $e_0(p)$, $0 \le p \le \alpha-1$. As this paper uses the repetitive process setting for analysis and design, the word pass will be used instead of trial from this point onwards.

Repetitive processes make a series of sweeps or passes through a set of dynamics defined over a finite interval or duration (Rogers et al., 2007). The pass profile is the name given to the output produced on each pass and once a pass is completed the process resets to the starting location and the next one commences. Moreover, the previous pass profile acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. This is the source of the unique control problem where the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.