Systems & Control Letters 64 (2014) 36–42

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

2D systems based robust iterative learning control using noncausal finite-time interval data ☆

Błażej Cichy [a,*], Krzysztof Gałkowski [a], Eric Rogers [b]

[a] Institute of Control and Computation Engineering, University of Zielona Góra, ul. Podgórna 50, 65-246 Zielona Góra, Poland

[b] Electronics and Computer Science, University of Southampton, Southampton SO17 1BJ, UK

ARTICLE INFO

Article history:
Received 3 April 2013
Received in revised form
18 July 2013
Accepted 6 November 2013
Available online 14 December 2013

Keywords:
Iterative learning control
Robust control
2D systems
Noncausal data

ABSTRACT

This paper uses a 2D system setting in the form of repetitive process stability theory to design an iterative learning control law that is robust against model uncertainty. In iterative learning control the same finite duration operation, known as a trial over the trial length, is performed over and over again with resetting to the starting location once each is complete, or a stoppage at the end of the current trial before the next one begins. The basic idea of this form of control is to use information from the previous trial, or a finite number thereof, to compute the control input for the next trial. At any instant on the current trial, data from the complete previous trial is available and hence noncausal information in the trial length indeterminate can be used. This paper also shows how the new 2D system based design algorithms provide a setting for the effective deployment of such information.

1. Introduction

Many systems complete the same finite duration operation over and over again, where each execution is known as a trial and the duration the trial length. The exact sequence is that on completion of each trial, the system resets to the starting location and the next one begins. A common application is replicated by a robot undertaking a pick and place task over and over again, i.e., collect an object from a specified location, transfer it over a finite duration, deposit it at a fixed location or onto a moving conveyor, return to the starting location and then repeat this sequence of operations. In this paper, the notation used is $y_k(p)$, $0 \leq p \leq \alpha - 1$, $k \geq 0$, where y is the vector or scalar variable under consideration, $\alpha < \infty$ is the number of samples along the trial length and k is the trial number. Also if $y_{ref}(p)$, $0 \leq p \leq \alpha - 1$, is a given reference trajectory for the output, which is assumed to be a member of the signal space chosen for the output of the controlled system, $e_k(p) = y_{ref}(p) - y_k(p)$ is the error on trial k and in iterative learning control (ILC) the novel feature is the use of the previous trial error in the computation of the control input applied on the next trial, with a generalization to higher-order ILC where the errors from a finite number $l > 1$ of previous trials are used.

Since its inception, widely credited to [1], ILC has seen extensive developments from theory through to experimental benchmarking and actual applications. The survey papers [2,3] are starting points for the literature and these together with subsequent publications show a wide range of applications from industrial robotics to residual vibration suppression, microelectronics fabrication, process control and recently a technology transfer to healthcare in the form of robotic-assisted upper limb stroke rehabilitation. In this latter application [4] the patient makes repeated attempts to follow a reference trajectory replicating daily living tasks, such as reaching out to a cup across a table top, assisted by a robot and with electrical stimulation applied to the relevant muscles. At the end of each trial, the patient's arm is returned to the starting location and ILC used to compute the electrical stimulation to be applied on the next trial based on the previous trial error. If the patient is improving with increasing trial number then his/her voluntary effort should increase and the applied stimulation decrease and this has been confirmed in clinical trials.

In ILC, all previous trial data are available before the next trial begins and hence there is not a requirement that only causal in p previous trial data is used in the computation of the current trial input, i.e., the control applied at $p \in [0, \alpha]$ on trial k can use previous trial data at $p = p + 1, \ldots, \alpha$. The ability to use the noncausal information is a novel feature of ILC and many successful implementations use the special case of phase-lead where a term in $e_k(p + \lambda)$, $\lambda > 0$ is used to form the control input $u_{k+1}(p)$ on the next trial, where $\lambda = 1$ is common. In such cases, one alternative to consider is the inclusion of a weighted sum of previous trial error phase-lead terms or a weighted sum of such terms and ILC

☆ This work is partially supported by the National Science Centre in Poland, grant No. 2011/01/B/ST7/00475.

* Corresponding author. Tel.: +48 68 328 4733.
E-mail addresses: b.cichy@issi.uz.zgora.pl (B. Cichy), k.galkowski@issi.uz.zgora.pl (K. Gałkowski), etar@ecs.soton.ac.uk (E. Rogers).

0167-6911/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.sysconle.2013.11.002

lag terms $e_k(p-\beta)$, $\beta > 0$. The aim of this paper is to provide the basis of ILC design where the use of such information can be evaluated as a necessary step toward implementation.

If the along the trial dynamics are discrete then one setting for ILC design is, since the trial length is finite, to define super-vectors for the variables. For example, let the system be single-input single-output for the ease of presentation with a natural extension to the vector case. Then the error super-vector is $E_k = \begin{bmatrix} e_k(0) & e_k(1) & \dots & e_k(\alpha-1) \end{bmatrix}^T$ and the ILC error dynamics can hence be written as a system of linear difference equations in k of the form $E_{k+1} = HE_k$.

Due to the finite trial length, error convergence in k can occur even if the system has an unstable state matrix. The solution via lifting design is to first design a stabilizing feedback control law and then apply ILC to the resulting controlled system. Also for robust control based on norm bounded or polytopic uncertainty, the entries in the matrix H will contain products of the matrices describing the uncertainty and this makes the analysis significantly more involved. Moreover, robustness analysis in the frequency domain is approximated as ILC controllers operate on a finite time interval, the trial length. Use of the Fourier transform on the infinite time interval will give a linear time-invariant control law but over the finite trial length errors may result in the initial part of the transient response; an example to support this fact is given in [5]. A robust H_∞ based ILC design with noncausal finite trial length is also given in [5] but is somewhat involved and choosing exactly what noncausal data to include is also lacking somewhat in transparency.

An alternative to the lifted model analysis is to exploit the natural 2D system structure of ILC where one direction of information is from trial-to-trial, indexed by the subscript k, and the other along a trial, indexed by p. The first results on 2D system based ILC analysis is credited to [6] where a Roesser state-space model was used. Repetitive processes [7] are another class of 2D systems where information propagation in p is over a finite duration and therefore a more obvious setting for analysis. The previous work on ILC laws designed using repetitive process control theory with experimental verification includes [8,9]. This setting also extends to differential dynamics and for robust control studies avoids products of matrices describing the uncertainty assumed. This paper will show that the repetitive process setting enables control law design where a weighted sum of noncausal and/or causal in p is used and hence, for a given example, different combinations of previous trial terms can be considered in the search for a design that meets the performance specifications, with an extension to robust control. The relative merits of this design method are also discussed.

Throughout this paper, the null and identity matrices with the required dimensions are denoted by 0 and I respectively. Also $M \succ 0$ ($\prec 0$) denotes a real symmetric positive (negative) definite matrix and $X \preceq Y$ is used to represent the case when $X - Y$ is a negative semi-definite matrix. Finally, diag $\{\cdots\}$ denotes a block diagonal matrix.

2. System representation and design

2.1. Discrete linear repetitive processes

Discrete linear repetitive processes evolve over the subset of the positive quadrant in the 2D plane defined by $\{(p,k) : 0 \le p \le \alpha - 1, k \ge 0\}$, and the basic state-space model for their dynamics has the following form [7]:

$$\begin{aligned} \tilde{x}_{k+1}(p+1) &= A\tilde{x}_{k+1}(p) + B\tilde{u}_{k+1}(p) + B_0\tilde{y}_k(p), \\ \tilde{y}_{k+1}(p) &= C\tilde{x}_{k+1}(p) + D\tilde{u}_{k+1}(p) + D_0\tilde{y}_k(p). \end{aligned} \tag{1}$$

In this model on pass k, $\tilde{x}_k(p) \in \mathbb{R}^n$ is the state vector, $\tilde{y}_k(p) \in \mathbb{R}^m$ is the pass profile vector, and $\tilde{u}_k(p) \in \mathbb{R}^r$ is the vector of control inputs. The simplest form of boundary conditions is $\tilde{x}_{k+1}(0) = d_{k+1}$, $k \ge 0$, where the $n \times 1$ vector d_{k+1} has known constant entries and $\tilde{y}_0(p) = f(p)$, where $f(p)$ is an $m \times 1$ vector whose entries are known functions of p.

Repetitive process has their origins in the coal mining industry where in longwall coal cutting the machine rests on the previous pass profile, which is the height of the stone/coal interface above some datum line, during the production of the current pass profile. It is therefore unrealistic to assume that at instance p on the current pass the only previous pass profile contribution comes from the same instance on the previous pass. An alternative model is

$$\begin{aligned} \tilde{x}_{k+1}(p+1) &= A\tilde{x}_{k+1}(p) + B\tilde{u}_{k+1}(p) + \sum_{i=-w_l}^{w_h} B_i\tilde{y}_k(p+i), \\ \tilde{y}_{k+1}(p) &= C\tilde{x}_{k+1}(p) + D\tilde{u}_{k+1}(p) + \sum_{i=-w_l}^{w_h} D_i\tilde{y}_k(p+i), \end{aligned} \tag{2}$$

where w_l and w_h are positive integers and the boundary conditions applied in this paper are those for (1), with the additional assumption that

$$\tilde{y}_k(p) = 0, \quad p \in \{-w_l, \dots, -1\} \cup \{\alpha, \dots, \alpha + w_h - 1\}. \tag{3}$$

Setting $w_l = 0$ and $w_h = 0$ recovers the previous model. In this alternative model on pass k and instance p the previous pass profile contribution is modeled as a linear sum of those at $0 \le p - w_l \le p \le p + w_h \le \alpha - 1$. The resulting model structure has no 2D discrete linear system state-space model interpretation, such as the Roesser model [10]. This paper applies the stability theory for this repetitive process model to ILC design.

2.2. Iterative learning control (ILC)

The systems considered are assumed to be modeled by linear time-invariant dynamics in the discrete domain with state-space model triple $\{A_d, B_d, C_d\}$. In ILC analysis, notation for the trial dependence is required. In this paper the subscript k is used, giving the following description:

$$\begin{aligned} x_k(p+1) &= A_d x_k(p) + B_d u_k(p), \, x_{k+1}(0) = d_{k+1} \\ y_k(p) &= C_d x_k(p), \quad 0 \le p \le \alpha - 1, \end{aligned} \tag{4}$$

where $k \ge 0$ is the trial number, $\alpha < \infty$ is the number of samples along the trial, $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the output vector, $u_k(p) \in \mathbb{R}^l$ is the control input vector and the entries in d_{k+1} are known constants.

Let $y_{ref}(p) \in \mathbb{R}^m$ denote the reference vector, which is assumed to be a member of the signal space chosen for the output of the controlled system, and hence the error on trial k is

$$e_k(p) = y_{ref}(p) - y_k(p). \tag{5}$$

A commonly used ILC strategy constructs the current trial input as sum of that used on the previous one plus a corrective term, i.e.,

$$u_{k+1}(p) = u_k(p) + \Delta u_{k+1}(p), \quad k \ge 0, \tag{6}$$

where $\Delta u_{k+1}(p)$ denotes the correction term.

Introduce, for analysis purposes only, the vector

$$\eta_{k+1}(p+1) = x_{k+1}(p) - x_k(p).$$

Suppose also that in the ILC law (6)

$$\Delta u_{k+1}(p) = K_1\eta_{k+1}(p+1) + K_2 e_k(p+1), \tag{7}$$