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Unconditional Maximum Likelihood Approach for Localization of Near-Field Sources: Algorithm and Performance Analysis*

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Abstract: Localization of near-field sources requires sophisticated estimation algorithms. In this paper, we propose an unconditional maximum likelihood method for estimating direction of arrival and angle parameters of near-field sources. However, the calculation of maximum likelihood estimation from the corresponding likelihood function results in difficult nonlinear constraint optimization problems. We therefore employed an Expectation/Maximization (EM) iterative method for obtaining maximum likelihood estimates. The most important feature of the EM algorithm is that it decomposes the observed data into its components and then estimates the parameters of each signal component separately providing computationally efficient solution to resulting optimization problem. The performance of the unconditional maximum likelihood location estimator for the near-field sources is studied based on the concentrated likelihood approach to obtain Cramér-Rao bounds. Some insights into the achievable performance of the conditional maximum likelihood algorithm is obtained by numerical evaluation of the Cramér-Rao bounds for different test cases.

Keywords: Maximum Likelihood estimation, Antenna arrays, Near-field source localization

1. Introduction

The problem of localizing multiple narrow band sources by a passive sensor array is common to diverse applications including radar, sonar, communication, seismology and electronic surveillance [1–3]. However, the majority of the localization methods deals with the case in which the source is assumed to be in the far-field of the array [1, 2]. That is, the source is assumed to be at an infinite distance from the array, and hence, the waves emitted by the sources can be considered as plane waves. Thus each source location can be characterized by only the azimuth (bearing). When the sources are located close to the array (i.e., near-field), the inherent curvature of the waveforms can no longer be neglected. Therefore, the spherical wavefronts in the near-field scenario must be considered and the location of each source have to be parametrized in terms of the direction of arrival (DOA) and range [3–6].

The localization of the near-field sources is in general nontrivial, since the localization of near-field sources requires estimation of the azimuth together with the range parameters. It is therefore necessary to derive more sophisticated localization algorithms for near-field sources. 7 years ago, a total least squares ESPRIT like algorithm based on fourth order cumulants has been developed [3]. A high resolution algorithm that uses only second order statistics of the array outputs was introduced in [4]. Due to many attractive properties of maximum likelihood (ML) estimation methods such as consistency, asymptotic unbiasedness and asymptotic minimum variance, we concentrate on maximum likelihood (ML) methods for localization of near-field sources. Regarding the assumption on the narrow-band source signals, there are two different types of models. These two models lead to corresponding ML solutions. The models are

i. Conditional model (CM) which assumes the signals to be unknown but deterministic (i.e., the same in all realizations)
ii. Unconditional Model (UM) which assumes the signals to be random.

ML methods corresponding to the signal models (i) and (ii) are termed conditional ML and unconditional ML respectively. Expectation/Maximization (EM) based conditional ML (signal model (i)) near-field location estimators have been studied in [5]. The goal of the present paper is to provide an unconditional ML approach for the estimation of the DOA and range parameters of near-field sources. However, the calculation of the ML estimation from the corresponding likelihood function for the unconditional case results in a difficult nonlinear constrained optimization problem, which must be solved iteratively. We therefore employed the EM iterative method for obtaining a ML estimator. The most important feature of the algorithm is that it decomposes the observed data into its signal components and then estimates the parameters of each signal component separately.

The asymptotical performance of an unconditional ML near-field localization technique is analyzed via the Cramér Rao Bounds (CRB) as benchmark for evaluating the performance of actual estimators. The technique for the derivation of CRB's used herein relies on a simple modification of the log-likelihood function. The log-likelihood function is modified by replacing the nuisance parameters with their ML estimates. It therefore avoids

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the process of explicitly calculating and invert the entire Fisher Information Matrix (FIM) [9]. We substitute the ML estimates of the array observation covariance matrix to obtain concentrated covariance matrix. We then calculated the CRB's for near-field source location parameters as in [9].

The structure of the paper is as follows. The matrix form of the near-field signal model is presented in Section 2. Section 3 contains a detailed derivation of the the unconditional ML estimator and the corresponding assumptions on the signal model. Moreover, the iterative EM algorithm for obtaining ML estimates from the corresponding unconditional cost function is introduced. In Section 4, the performance of the unconditional ML method is evaluated based on the concentrated ML approach to obtain CRB's. In Section 5, simulation results are presented. Finally, some conclusions of the work are reported.

Notations used in this paper are standard. Symbols for matrices (in capital letter) and vector (lower case) are in boldface. $(\cdot)^T$, $(\cdot)^H$ denote transpose and complex conjugate transpose respectively. $\odot$ denotes the element-wise matrix products (Schur-Hadamard product) whereas $\otimes$ denotes the Kronecker product. $\mathbf{I}$ is the identity matrix with an appropriate dimension.

2. Signal model

Consider d narrow-band, co-channel sources with wavelength λ, impinging on an M element array. Each source is located at angle θ_i and at range r_i, $1 \le i \le d$, relative to the reference sensor with index $'0'$. The array is assumed to be uniform, linear, and consisting of omnidirectional sensors with inter-element spacing Δ, and sensor indices $k \in K = \{k_{min}, \cdots, k_{max}\}$.

Then, in the presence of additive noise, the output of the k^{th} sensor can be expressed as

$$x_k(t_n) = \sum_{i=1}^{d} s_i(t_n) e^{j\left(\mu_i k + \zeta_i k^2\right)} + n_k(t_n), \quad 1 \le t_n \le N \quad (1)$$

where $n_k(t_n)$ is the sensor noise, $s_i(t_n)$ is the complex envelope of the i^{th} source [1, 3, 5]. The effect of spherical curvature of the wavefronts in the near-field scenario is approximated by including an extra term in phase via second-order polynomial phase approximation. The first phase component corresponds to the far-field phase $\mu_i = -\frac{2\pi\Delta}{\lambda}\sin\theta_i$. $\zeta_i = \frac{\pi\Delta^2}{\lambda r_i}\cos^2\theta_i$ is the extra term included to approximate the effect of near-field source and is a nonlinear function of azimuth and range.

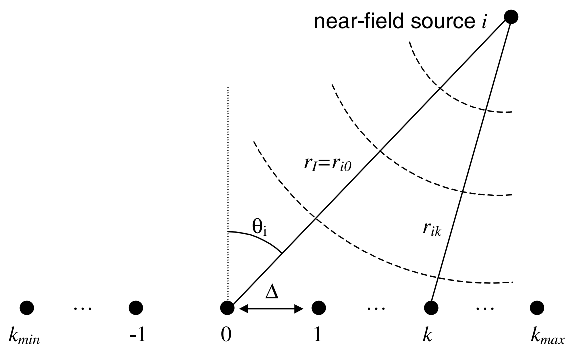

Fig. 1. Near-field scenario with a uniform linear array.

Letting $\boldsymbol{\mu} = [\mu_1, \cdots, \mu_d] \in \mathbb{R}^{d\times 1}$ and $\boldsymbol{\zeta} = [\zeta_1, \cdots, \zeta_d] \in \mathbb{R}^{d\times 1}$ denote the vector of near-field location parameters and $\mathbf{x}(t_n) = [x_{k_{min}}, \cdots, x_{k_{max}}]$ denote a vector from the sensor outputs, the matrix vector formulation of (1) can be written as follows;

$$\mathbf{x}(t_n) = \mathbf{A}(\boldsymbol{\mu}, \boldsymbol{\zeta})\mathbf{s}(t_n) + \mathbf{n}(t_n), \qquad 1 \le t_n \le N \qquad (2)$$

where $\mathbf{s} \in \mathbb{C}^{d\times 1}$ in the source signal vector, $\mathscr{s} = [\mathbf{s}^T(1), \cdots, \mathbf{s}^T(N)]^T \in \mathbb{C}^{Nd\times 1}$, $\mathbf{A}(\boldsymbol{\mu}, \boldsymbol{\zeta}) = [\mathbf{a}(\mu_1, \zeta_1), \cdots, \mathbf{a}(\mu_d, \zeta_d)] \in \mathbb{C}^{M\times d}$ is the array steering matrix. The expression for the column vector $\mathbf{a}(\mu_i, \zeta_i)$ of $\mathbf{A}(\boldsymbol{\mu}, \boldsymbol{\zeta})$ is in the following form

$$\mathbf{a}(\mu_i, \zeta_i) = \begin{bmatrix} e^{j\left(k_{min}\mu_i + k_{min}^2\zeta_i\right)} \\ \vdots \\ 1 \\ e^{j(\mu_i+\zeta_i)} \\ e^{j(2\mu_i+4\zeta_i)} \\ \vdots \\ e^{j\left(k_{max}\mu_i + k_{max}^2\zeta_i\right)} \end{bmatrix}. \qquad (3)$$

The problem of estimating the near-field source location parameters $\{\boldsymbol{\mu}, \boldsymbol{\zeta}\} = \{(\mu_1, \zeta_1), \cdots, (\zeta_d, \mu_d)\}$ from N snapshots $\mathscr{x} = [\mathbf{x}^T(1), \cdots, \mathbf{x}^T(N)]^T$ is the main concern of the paper.

3. Unconditional ML estimator

In this section we derive the unconditional ML estimator for the problem defined above. To describe the unconditional ML estimator's derivation, we made the following assumptions on the signal model (1):

AS1: The source signal $\mathbf{s}(t_n)$ is temporally and spatially uncorrelated circular complex Gaussian random process with zero-mean and nonsingular unknown covariance matrix $\mathbf{K_s}$,

$$E\left[\mathbf{s}(t_n)\mathbf{s}^H(t_m)\right] = \mathbf{K_s}\delta_{t_n,t_m}$$
$$E\left[\mathbf{s}(t_n)\mathbf{s}^T(t_m)\right] = \mathbf{0} \quad \text{for all } t_n \text{ and } t_m \,. \qquad (4)$$

where δ_{t_n,t_m} is the Kronecker delta ($\delta_{t_n,t_m} = 1$ if $t_n = t_m$ and 0 otherwise), $(\cdot)^H$ is the conjugate transpose and $(\cdot)^T$ is the transpose of a matrix.

AS2: The additive noise vector $\mathbf{n}(t_n)$ is temporally and spatially uncorrelated circular complex Gaussian process with zero-mean and standart derivative σ^2 as

$$E\left[\mathbf{n}(t_n)\mathbf{n}^H(t_m)\right] = \sigma^2\mathbf{I}\delta_{t_n,t_m} \qquad (5)$$

$$E\left[\mathbf{n}(t_n)\mathbf{n}^T(t_m)\right] = \mathbf{0} \quad \text{for all } t_n \text{ and } t_m \,. \qquad (6)$$