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Robust iterative learning for high precision motion control through $\mathcal{L}_1$ adaptive feedback

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ABSTRACT

The diversity of precision motion control applications and their demanding design specifications pose a large array of control challenges. Hence, precision motion control design relies on a variety of advanced control strategies developed to cope with specific problems present in control theory. A popular feedforward control technique for repetitive systems is iterative learning control (ILC). While ILC can decrease tracking errors up to several orders of magnitude, the achievable performance is limited by dynamic uncertainty. We propose the combination of $\mathcal{L}_1$ adaptive control ($\mathcal{L}_1$ AC) and linear ILC for precision motion control under parametric uncertainties. We rely on the adaptive loop to compensate for parametric uncertainties, and ensure that the plant uncertainty is sufficiently small so that an aggressive learning controller can be designed on the nominal system. We exploit the closed loop stability condition of $\mathcal{L}_1$ AC to design simple, robust ILC update laws that reduce tracking errors to measurement noise for time varying references and uncertainties. We demonstrate in simulation that the combined control scheme maintains a highly predictable, monotonic system behavior; and achieves near perfect tracking within a few trials regardless of the uncertainty present.

1. Introduction

Iterative learning control (ILC) is a feedforward control strategy for systems that execute the same task repeatedly over a finite time horizon [1]. ILC is based on the idea that the tracking performance of such systems can be improved by using information from previous trials. Contrary to other learning type control strategies (e.g. adaptive control, neural networks, repetitive control), ILC modifies the input signal rather than the controller [2]. In a way, ILC is a form of feedback control over the iteration domain. Consistent with this property, iterative learning controllers offer simplicity, robustness and fast convergence to iteration domain equilibria with performance improvements up to several orders of magnitude over conventional control strategies.

One of the essential challenges that motivates the field of ILC is dynamic uncertainty. Much as in feedback control, the main approaches for mitigating uncertainty can be roughly classified as robust or adaptive methods. Considerable research has been done on the synthesis of ILC algorithms that are robust to exogenous disturbances, stochastic effects, interval uncertainties, and high frequency modeling uncertainties (see [1,3] and references therein). Refs. [4–6] provide good examples of $\mathcal{H}_\infty$ methods for finite and infinite horizon cases; an area in which much work has been done. In [7], the combination of $\mathcal{H}_\infty$ feedback control with ILC was analyzed, with the premise of bandwidth separated repetitive and nonrepetitive exogenous signals. One particular example that underlines parametric uncertainties from a robustness perspective is [8], in which stability of ILC to interval uncertainties in the impulse response is evaluated. The drawback to these methods is that while ILC convergence is guaranteed within the prescribed set of uncertainties, performance is often limited due to conservative designs. Additionally, the sensitivity of robust learning controllers to variations in the uncertainties is still an open question.

Parametric uncertainties have similarly been studied extensively in the adaptive ILC setting with special attention to the application area of robotics, wherein iterative estimation schemes were used to augment the feedback controllers using Lyapunov like methods [9,10]. Iterative estimation was also used to reduce the model tracking error and improve transient response in model reference adaptive control (MRAC) [11–13]. Other works showed how adaptive feedback control methods can be extended to ILC in a straightforward way [14], and

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proved universal adaptive ILC laws for single-input single-output (SISO) linear time invariant (LTI) systems with nonzero first Markov parameters [15]. While the adaptive nature of these systems signify high performance and reduced sensitivity to parametric variations, the robustness of adaptive ILC to unmodeled dynamics may be questionable, analogous to adaptive feedback control [16,17].

Most of the fundamental limitations and trade-offs of control theory can be observed to a greater extent in precision motion control due to complex, demanding design specifications. Key issues in the control of precision positioning systems include robustness to parameter variations, unmodeled high frequency dynamics, and the bandwidth-precision trade-off [18]. More complex process modeling can mitigate uncertainty issues to an extent, but this becomes unfeasible as complexity increases, specifically due to the fact that certain information about the process, such as external loads and/or parameters that are sensitive to exogenous effects, cannot be known a priori. Although adaptive feedback methods provide a good solution to the problem of *robustness to parametric variation* and increase precision, this often comes at the expense of *reduced robustness to unmodeled dynamics* [17] as fast estimation, which is desired from a performance standpoint, leads to high gain feedback. This problem essentially boils down to the fact that conventional adaptive control ignores Bode's sensitivity integral [19,20], also known as the *waterbed effect*, by compensating for uncertainties throughout the whole frequency spectrum. Similarly, while ILC extends the *available bandwidth* [20] of the control channel for repetitive systems, thereby alleviating the bandwidth-precision trade-off, the achievable reduction in errors and monotonicity on the iteration axis depends largely on the level of uncertainty in the feedback stabilized plant.

To address these issues, this work combines conventional ILC with $\mathcal{L}_1$ adaptive feedback control, and is an extension of our previous work in [21–23]. $\mathcal{L}_1$ adaptive control ($\mathcal{L}_1$ AC) is a recent MRAC paradigm that bridges the gap between adaptive and robust control with a priori known, quantifiable transient response *and* robustness bounds [17]. The idea of combining ILC with $\mathcal{L}_1$ AC was first introduced in [21], wherein the adaptive loop was utilized to keep the plant sensitivity close to its nominal value for performance improvement through learning. Despite the displayed advantages of $\mathcal{L}_1$ AC over linear feedback, a trade-off was observed between the closed loop bandwidth and learning performance. More precisely, it was seen that higher closed loop bandwidths resulted in slower convergence and larger converged errors in the iteration domain. To resolve this problem, we proposed the augmentation of the $\mathcal{L}_1$ AC architecture with an arbitrary feedforward signal to accommodate learning, leading to an adaptation that considers changes in the nominal system behavior due to learning [22]. The resulting $\mathcal{L}_1$ AC-ILC ($\mathcal{L}_1$-ILC) scheme had predictable performance in both the time and iteration domains: The feedforward augmented closed loop preserved the a priori known quantifiable transients from $\mathcal{L}_1$ AC theory, and the learning controller displayed similar convergence behavior regardless of the uncertainty present in the system. It was also seen that increasing feedback bandwidths resulted in decreasing effects of uncertainty in the iteration domain, with faster convergence and lower converged errors. In [23], we presented design guidelines and showed the performance gains of the modified scheme over linear output feedback on a large range nanopositioner via simulation. The main differences between this work and our previous work include:

1. A generalized approach to $\mathcal{L}_1$-ILC for different classes of linear systems through vector space methods.
2. Extension of the robust monotonic learning convergence results to time varying parametric uncertainties.
3. Design guidelines for the $\mathcal{L}_1$-ILC scheme that link feedback-learning filter designs to classical control ideas, and show how the $\mathcal{L}_1$ AC stability condition can be satisfied for a given system.
4. Validation of the performance improvements of the proposed scheme in comparison with an LTI feedback based ILC, through extensive simulations on a precision positioning system subject to time varying parametric uncertainties.

Our work differs from the existing literature in several ways: First, as we have mentioned, previous work on adaptive methods in learning have focused on *adaptive ILC*, wherein adaptive learning laws are considered with or without adaptive feedback. Second, adaptive feedback has not been used in a robust ILC setting before. Third, although the idea of combining ILC with advanced feedback methods to achieve better performance is not new, to the best of our knowledge, the combination of conventional ILC with adaptive feedback has not been employed before.

In this paper, we demonstrate how ILC algorithms can be combined with $\mathcal{L}_1$ AC schemes to achieve robust, high precision motion control. We present feedforward augmented $\mathcal{L}_1$ AC architectures for state and output feedback cases (see Figs. 2 and 5)) to accommodate parallel ILC signals and show how this preserves the a priori known $\mathcal{L}_1$ AC transient bounds. We explain how these bounds, which imply arbitrary close tracking of *linear* reference models in the time domain, can be exploited for learning purposes in the iteration domain. We then show how the $\mathcal{L}_1$ AC stability condition relates directly to the robust monotonic convergence conditions of LTI learning laws, and how robust ILC algorithms can be designed in a simple, straightforward manner for different $\mathcal{L}_1$ AC architectures.

The rest of the paper is organized as follows. Section 2 introduces some preliminaries for clarity of exposition. Section 3 gives a brief introduction to $\mathcal{L}_1$ AC and ILC, and presents our proposed method for the state feedback case. Section 4 extends the results to time varying uncertainties in output feedback. Simulation results are given in Section 5. Section 6 gives concluding remarks and summarizes our findings. For a streamlined presentation, we give certain intermediate results in Appendix A, proofs of our main results in Appendix B and several auxiliary variables in Appendix C.

2. Notation and preliminaries

Throughout the paper, we use time and frequency domain representations interchangeably for signals. For example, $f(s)$ denotes the Laplace transform of the signal $f(t)$. We denote systems and matrices with upper case letters. We represent signals and vectors with lower case letters. We use script letters to distinguish linear operators in general from their matrix and transform representations (e.g. $\mathcal{F}$ instead of $F(s)$). We take $\mathbb{R}$ to represent the set of real numbers and $\mathbb{R}^+$ the set of positive real numbers. We choose $\mathbb{C}$ to denote complex numbers. We take $\mathbb{I}$ to be the identity matrix of appropriate size and $\mathcal{I}$ to be the identity operator in the relevant space. We use $\lambda_{max}(\cdot)$ and $\lambda_{min}(\cdot)$ to denote the maximum and minimum eigenvalues of a positive definite matrix, respectively. We take $\|\cdot\|_p$ for $p \in [1, \infty]$ as the standard vector and induced p norm. We use $\mathcal{F}^{-R}$ and $\mathcal{F}^{-L}$ for the right and left inverses of an operator $\mathcal{F}$, respectively; and F^T for the transpose of a matrix F.

In the rest of the section, we collect several definitions and facts from systems theory pertinent to our discussion.

Definition 1. For any $p \in [1, \infty), \mathcal{L}_p^n$ is defined as the space of all piecewise continuous $f : \mathbb{R} \to \mathbb{R}^n$ such that $\|f\|_{\mathcal{L}_p} \triangleq \left(\int_{-\infty}^{\infty} \|f(t)\|^p dt\right)^{1/p} < \infty$, where $\|\cdot\|$ is any standard vector

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