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PERFORMANCE ANALYSIS AND OPTIMIZATION OF A THERMALLY NON-SYMMETRIC ANNULAR FIN

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ABSTRACT

Considering thermally non-symmetric convective boundary conditions, optimum dimensions of an annular fin which has a rectangular cross-section are investigated. Two-dimensional heat diffusion equation is solved analytically to obtain temperature distribution and heat transfer rate. In this work, fin volume is fixed to obtain the dimensionless geometrical parameters of the fin with maximum heat transfer rates. The optimum geometry which maximizes the heat transfer rate for a given fin volume has been found employing NCONF routine in the IMSL Library. The derived condition of optimality gives an open choice to the designer.

Introduction

Finned surfaces have been in use for a long period as a heat dissipation mechanism. An extensive review is seen on this topic [1]. Technology has led to a demand for high-performance, light-weight and compact heat transfer components. In order to accommodate the demand, finned surfaces are used to increase the heat transfer rate between a primary surface and the surrounding fluid in heat exchange devices. Thus, optimization of the design of fins is of significant importance. The optimization of fins is generally based on two approaches: one is to minimize the volume or mass for a given amount of heat dissipation and the other is to maximize the heat dissipation for a given volume or mass [2-4]. For purely conductive and convective fins, the criterion for the optimal problem was first proposed by Schmidt [5].

Several papers [6-8] are devoted to finding the shape of fins that would minimize the volume for a given amount of heat dissipation. However, all of these studies consider negligible heat transfer from the tip, because their fins have sharp tips. Apparently, the sharp-tip fin design has the disadvantage that the resulting profiles are too difficult to manufacture. Several other studies [9-11] conducted to find the dimensions of a constant thickness straight fin that would maximize the heat dissipation for a given volume These studies, however, invoked the assumption of insulated tips, and furthermore, they used one-dimensional heat transfer approach. It has been shown that the one-dimensional approach is convenient, but may be in error for certain physical conditions [2, 12-14]. These studies, however, invoked the assumption of insulated tips, and furthermore, they used one-dimensional heat transfer approach. It has been shown that the one-dimensional approach is convenient, but may be in error for certain physical conditions [2, 12-14]. With regard to the applicability of the one dimensional model for heat conduction in the fins, some arguments based either on the fin effectiveness [15] or on evaluation of the largest transverse variation of fin temperature [16] are available to demonstrate that the one dimensional assumption is valid only for a transverse Biot number much less than unity, $Bi<<1$, a statement usually known as the 'validity criterion' [17]. Aziz and Lunardini [18] concluded that error on the dimensionless heat flow between the predictions of the one dimensional model and the correct two dimensional model, for a longitudinal rectangular fin, is approximately of 1% if $Bi=0.1$.

In this paper, optimization of annular fins which have non-symmetric boundary conditions is considered. The problem is to maximize the heat-transfer rate for a given fin volume. The optimization variables are fin thickness and ratio of outer radius to inner radius that will be called the fin ratio in the following sections. It is assumed that the predominant modes of heat transfer are conduction and convection, and the effect of radiation is ignored.

Problem Description and Analytical Solution

Consider an annular fin of a homogeneous material with rectangular profile is attached to a cylindrical surface of radius, a, with a base temperature of T_b, which is measured in excess to the ambient fluid temperature, T_∞. The geometry of the fin is shown schematically in Figure 1. Considering steady-state conditions and also assuming that the thermal conductivity is constant, the radiation from the surfaces is negligible, the heat generation effects are absent, and the convection heat transfer coefficients h_1, h_2 and h_3 are uniform over the surfaces; the equation for steady-state temperature distribution of the fin can be written as [19],

$$\frac{\partial^2 T'}{\partial r'^2}+\frac{1}{r'}\frac{\partial T'}{\partial r'}+\frac{\partial^2 T'}{\partial z'^2}=0 \qquad a<r'<b,\ 0<z'<c \tag{1}$$

$$-k\frac{\partial T'}{\partial z'}+h_1\left(T'-T'_\infty\right)=0 \qquad \text{for} \qquad z'=0 \tag{2a}$$

$$k\frac{\partial T'}{\partial z'}+h_2\left(T'-T'_\infty\right)=0 \qquad \text{for} \qquad z'=c \tag{2b}$$

$$T'=T_b \qquad \text{for} \qquad r'=a \tag{2c}$$