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# Event oriented system analysis

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**Abstract**

This article presents an attempt towards a probabilistic event oriented system analysis in engineering. Engineering systems are represented as either complete or incomplete systems of events and as compounds of various subsystems of events. The event oriented system analysis investigates important subsystems in engineering systems, such as operational modes and failure modes and their interrelations. The analysis is also applicable to engineering systems with various relations among the sets of events, such as mutually exclusive and inclusive sets. Further, the systems and subsystems are subjected to probability and uncertainty analysis. The system uncertainty analysis is based on entropy. General relations among the probability, uncertainty of the system and uncertainties of the subsystems are derived by using information theory. Specific mathematical aspects and available methods in the uncertainty modelling of systems and subsystems are summarised. Numerical examples confirm the relevance of the event oriented system analysis and indicate potential improvements in system design. 

*Keywords*: Engineering; Mechanics; System analysis; System of events; Probability; Reliability; Entropy; Information; Uncertainty

## 1. Introduction

Each object can be viewed as a system in different ways. An object is often a part or subsystem of a more complex system. The object itself can consist of many components and possibly of more subsystems. Systems in engineering are often viewed as objects of many discrete interacting components with uncertain capabilities. Moreover, systems are subjected to uncertain external demands. Sometimes, the components are grouped into a number of subsystems, each of them pertaining to some specific characteristic function of the system. Subsequently, many systems can be assumed to depend only on the current states of their components.

Complex engineering systems can be subjected to service modes and effects analysis in order to identify the events that can occur at the component, subsystem and system levels. The goals of such an analysis are to determine the effects of known operational and failure modes on the overall behaviour of the systems [3,8,11]. In addition to operational modes and effects analysis, semiquantitative and quantitative methods can be applied to predict the probabilities of safe operation or the accidents [10]. Redundancies [5,6] and robustness [4] can also be considered. Service modes and effects analysis is an essential step towards understanding complex systems without which reliability and uncertainty analysis cannot be performed.

The procedures presented in this article are applied in addition to the traditional system analysis for the solution of practical numerical examples in engineering. The aim is to demonstrate the usefulness of the event oriented system analysis as a tool for assessment of system performances and improvements of engineering system design with respect to system uncertainties.

## 2. Uncertainty modelling

There are many uncertainties concerning systems and various methods of analysis. However, in the usually adopted random variable model, the uncertainties of the components are due to statistically uncertain capabilities regarding the geometry, the material properties, the workmanship, different uncertain demands, operational conditions and loads, as well as modelling and subjective uncertainties. The probabilistic system analysis is based on the application of probability theory to basic stochastic events, which can be defined by random design variables. Such an analysis provides system reliability or system failure probability using the usually set algebra.

The traditional probabilistic approach to discrete engineering systems (mechanical, structural, electrical, aerospace, nuclear, marine, etc.), with uncertain capabilities and operating under uncertain conditions, takes into account the random physical and technical characteristics of the components of the system and the stochastic environmental effects.

Moreover, systems may be considered at another level. In the event oriented system analysis, a system is defined not

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**Nomenclature**

| | |
|---|---|
| $A_i$, $E_i$ | Random events in general |
| $H(\cdot)$ | Entropy of a system of events |
| $H^1(\cdot)$ | Entropy of a system of events of order one |
| $N$, $n$ | Numbers of systems and of subsystems |
| $N_o$, $N_f$ | Number of operational events and of failure events |
| $p(\cdot)$, $p_i$ | Probabilities of random events, (sub)systems |
| $\mathscr{S}$ | System of events in general |
| $\mathscr{S}'$ | System of subsystems of events |
| $\mathscr{S}_i$ | Subsystem of events in general |
| $\mathscr{O}$, $\mathscr{F}$ | Subsystems of operational and of failure events |

only by its physical and/or technical components, but also by its all, or at least known or important states. Primarily, the states of the engineering system can be considered as operational or inoperational. The majority of the states are observable, but there may be some unobservable, undefined, unknown or less important states as well. Some of the states can be in common with several system features. The states of a system are represented by a system and by subsystems of random events in different relations and on various levels.

The word "uncertainty" in the context of engineering system of events should be given a more precise meaning [14]. The notion of uncertainty, applied to a system of events, is an uncertainty in the objective sense due to the fact that actually several events are possible. It is not the uncertainty in the mind of observers concerning the outcomes of an experiment [13]. The uncertainty arises from the number of events and the unpredictability of the events or subsystems.

In connection with the notion of uncertainty, the concept of information has to be mentioned. The uncertainty diminishes with the reception of relevant information. The uncertainty with respect to outcome may be considered equal to the information furnished by the occurrence of this outcome. Thus, uncertainty can also be measured. Terminology often alternates. The concept of entropy in the information theory was first applied to transmission of various information. Later, it was extended to the probability theory and engineering systems, providing more comprehensive definitions of system characteristics by introducing system uncertainties. A strong connection also exists between the notion of entropy in thermodynamics and the information theory.

The basic idea in this article is to make use of Shannon's entropy or information [12] to assess the uncertainty of systems and subsystems. In addition to the basic definition of entropy given by Shannon and Weaver for complete systems, Renyi's entropy [13] can be used to assess the uncertainty of incomplete system of events. The entropy can be considered both as the measure of the uncertainty, which prevailed before the experiment was accomplished, and as a measure of the information expected from an experiment. The theorem about the entropy associated with the mixture of distributions [13], and the theorem about dependent systems [9] can be used to assess the uncertainty of subsystems.

Possible ambiguities in a complex system reliability assessment can be resolved by uncertainty analysis. Such problems arouse when, for example, systems with the same reliability but different probability distributions of a number of operational and failure modes are evaluated.

## 3. Uncertainty measures

The uncertainty of a single stochastic event $A$ with a known probability $p = p(A) \neq 0$ plays a fundamental role in the information theory. The entropy of a single stochastic event is defined as $E = H_1(p) = -\log_2 p(A)$ [15], and can be interpreted either as a measure of how unexpected the event was, or as a measure of the information yielded by the event [1].

More important than single stochastic events are the systems of events. Events are considered as abstract concepts and the relations among events are characterised axiomatically.

The algebraic structure of the set of event turns to the Boolean algebra [13].

A system of events: $E_1, E_2, \ldots, E_n$ is called a complete system of events if the following relations in the events space hold:

$$E_k \neq \varnothing \qquad (k = 1, 2, \ldots, n) \tag{1}$$

$$E_j E_k = \varnothing \qquad (\text{for } j \neq k) \tag{2}$$

$$E_1 + E_2 + \cdots + E_n = I \tag{3}$$

- The "$\varnothing$" in Eqs. (1) and (2) means an impossible event.
- The fact that $E_j$ and $E_k$ are exclusive is expressed in Eq. (2).
- Eq. (3) denotes that at least one of the events $E_k$, $k = 1, 2, \ldots, n$, occurs.
- The $I$ denotes a definite event.

The definitions of complete and incomplete systems of events in probability space imply the following:

- A system of events $E_k$, $k = 1, 2, \ldots, n$, is said to be complete if for $i \neq j$, $A_i A_j = \varnothing$, and if the occurrence of an event $E_k$ is "almost sure", i.e. if it has the property $p(\sum_k E_k) = \sum_k p(E_k) = 1$.

The definition also involves that one and only one event must occur in each trial.

- If some outcomes of an 'experiment' are not known, or their probabilities cannot be determined, or if only some