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TETHERED SATELLITE SYSTEM ANALYSIS (1) — TWO-DIMENSIONAL CASE AND REGULAR DYNAMICS

SHAOHUA YU†

Center for Space Science and Applied Research, Academia Sinica, P.O. Box 8701. Beijing 100080, People's Republic of China

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Abstract—The study on tethered satellite system (TSS) in two-dimensional in-planar motion is restricted in that the tether is assumed to be massless. The equations of motion are given in a spherical coordinate system to describe the magnitude (tether length) and direction angle of the position vector between the satellites. A length rate control algorithm is adopted, and the controlled motion of the directional angle by the algorithm will have a stable equilibrium state. The equilibrium state is a fixed point if the orbit of the base-satellite is circular, and a limit cycle if the orbit is elliptic. The value and stability of the equilibrium state are determined by the parameters of the control algorithm, and the bifurcation analysis is also given. Two typical TSS missions have been simulated.

1. INTRODUCTION

The concept of a tethered satellite system (TSS) for space shuttle mission was put forward in 1974 [1]. According to this concept, a base-satellite and a subsatellite are connected by a flexible tether cable. The subsatellite can be deployed from the base-satellite through a tether reeling mechanism, be station-kept and retrieved back to the base-satellite. Many projects of using TSS in space concerning microgravity, atmospheric probe, spacecraft maneuver, electrodynamic tether, solar power station, etc., have been proposed since then [2–5]. In this connection, innumerable literature have been published to investigate the dynamics and control of TSS [6–12]. Among them a very impressive job has been done by Beletsky and Levin [6], and Rupp has proposed a valuable tether tension control law [7], Ivanov and Sitarsky gave very interesting treatment of the dynamics of TSS, based on the qualitative methods of the nonlinear differential equations [8]. The world is witness of the joint NASA-ASI project on TSS-1 which has been flown or to be reflown to verify the control techniques. A detailed analysis of TSS-1 is given in [9].

The space flights so far are mainly the flights of single spacecraft or of multiple spacecraft tightly docked together like the space stations. The flight of an ensemble of the soft connected spacecraft like TSS is perhaps the next step of space era. The dynamics of such flight is more complicated and has not been sufficiently investigated. Therefore, it should not be surprised that the first flight of TSS-1 was not successful.

This paper is first of the three consecutive papers on TSS study by the author. The other two papers to be published are related to the chaotic dynamics in three-dimensional motion and the dynamics of massive tether system. The controlled motion of TSS is treated by using a single spherical coordinate system and an unique tether length rate control algorithm. Both nonlinear dynamic system theory and linearization techniques are extensively used in these papers to develop the methods and algorithms to analyze the dynamics and stability of TSS.

2. SIMPLIFIED MATHEMATICAL MODEL

The mathematical model of TSS may be classified into two groups of equations of motion with the common assumptions that the earth is a central gravitational body, and that the base-satellite is far more heavier than the subsatellite so that the motion of the base-satellite is roughly a keplerian one, providing a reference to the following discussions. No thrusters are assumed on the subsatellite, and the tether has neither bending nor torsion stiffness.

The first group is called the simplified mathematical model which deals with the massless and

†Tel.: +86-10-6256-9834; fax: +86-10-6256-9834.
E-mail address: yu_shao_hua@sina.com.cn (S. Yu).

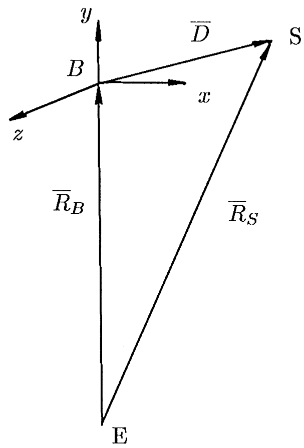

Fig. 1. System's geometry. Orbital coordinate system.

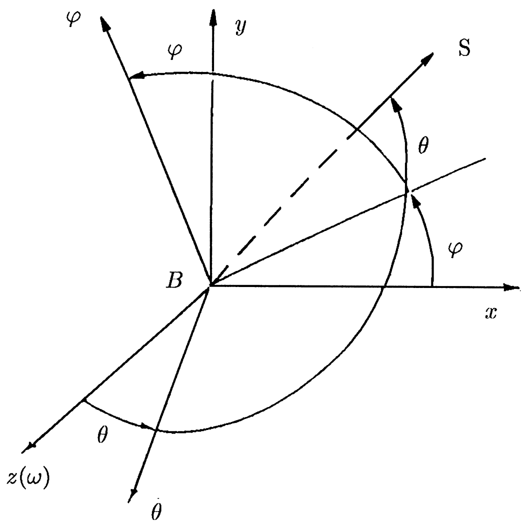

Fig. 2. Spherical coordinate system.

inextensible tether and is described by the ordinary differential equations; The second group of the mathematical models deals with the mass-distributed and extensible tether, which is described by a system of partial differential equations.

To derive the simplified mathematical model, it is helpful to neglect the electrodynamic force as well as the aerodynamic force on the tether so that a theoretical analysis would be possible which is very important to the problem.

Figure 1 shows the positions of the base-satellite (B), subsatellite (S), and Earth's center (E) forming the position vectors $\bar{R}_B$ and $\bar{R}_S$. An orbital coordinate system B_{xyz} with the origin at B, axis B_y aligned with the local upward vertical and axis B_z directed along the vector of the orbital angular velocity, is shown in Fig. 1. Figure 2 shows the spherical coordinate system $B_{s\varphi\theta}$ with axis B_s aligned with the line of sight BS. System $B_{s\varphi\theta}$ is obtained by rotating B_{xyz} through two angles φ and θ. φ is called the in-(orbital) plane or phase angle, θ is called the out-of-plane angle. The transfer matrix $L_{\theta\varphi}$ between the coordinate systems is

$$\{B_{s\varphi\theta}\} = L_{\theta\varphi} \cdot \{B_{xyz}\}$$

and

$$L_{\theta\varphi} = \begin{bmatrix} \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \end{bmatrix}. \quad (1)$$

The angular velocity vector $\bar{\omega}$ of the moving system $B_{s\varphi\theta}$ has the components

$$\omega_1 = -(\dot{v}+\dot{\varphi})\sin\theta, \quad \omega_2 = \dot{\theta}, \quad \omega_3 = (\dot{v}+\dot{\varphi})\cos\theta, \quad (2)$$

where v is the true anomaly of the base-satellite, $\dot{v}$ is the orbital angular velocity.

The simplified mathematical model describes the motion of the subsatellite with respect to the base-satellite, i.e. the motion of the relative position vector $\bar{D}$ (see Fig. 1)

$$\bar{D} = \bar{R}_S - \bar{R}_B$$

after two times differentiation

$$\ddot{\bar{D}} = \ddot{\bar{R}}_S - \ddot{\bar{R}}_B.$$

According to the assumption of central gravitation of the Earth, it can be written as

$$\ddot{\bar{R}}_S = -\frac{\mu}{R_S^3}\bar{R}_S - \frac{T}{m}\cdot\frac{\bar{D}}{D}, \quad \ddot{\bar{R}}_B = -\frac{\mu}{R_B^3}\bar{R}_B + \frac{T}{M}\cdot\frac{\bar{D}}{D},$$

where $\mu = 0.398602 \times 10^{15}$ m^3 s^{-2} is the Earth's gravitational constant, T is the tether tension force, m and M $(m \ll M)$ are the masses of the subsatellite and base-satellite, respectively. Therefore,

$$\ddot{\bar{D}} = -\frac{\mu}{R_S^3}\bar{R}_S + \frac{\mu}{R_B^3}\bar{R}_B - \frac{T}{m}\cdot\frac{\bar{D}}{D}. \quad (3)$$

The vector $\bar{D}$ and its derivatives in $B_{s\varphi\theta}$ are expressed as

$$\bar{D} = De_1, \quad \dot{\bar{D}} = \left(\frac{\partial\bar{D}}{\partial t}\right)_0 + \bar{\omega}\times\bar{D},$$

$$\ddot{\bar{D}} = \left(\frac{\partial\dot{\bar{D}}}{\partial t}\right)_0 + \bar{\omega}\times\dot{\bar{D}},$$

where e_1, e_2, e_3 are the unit vectors of system $B_{s\varphi\theta}$, $(\cdots)_0$ denotes the local differentiation in the system $B_{s\varphi\theta}$. In explicit form, it is

$$\begin{aligned}\ddot{\bar{D}} = {} & [\ddot{D} - D\dot{\theta}^2 - D(\dot{v}+\dot{\varphi})^2\cos^2\theta]e_1 + [D(\ddot{v}+\ddot{\varphi}) \\ & +2\dot{D}(\dot{v}+\dot{\varphi}) - 2D(\dot{v}+\dot{\varphi})\dot{\theta}\tan\theta]\cos\theta\cdot e_2 \\ & -[D\ddot{\theta} + 2\dot{D}\dot{\theta} + D(\dot{v}+\dot{\varphi})^2\cos\theta\sin\theta]e_3.\end{aligned}$$