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# Fuzzy feedback system analysis using transition matrices

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**Abstract**

An analytical characterization of fuzzy feedback systems based on transition matrices is carried out in this paper. The analysis faces both systems which use linear operators (sum and product) and those based on the max–min operators. We focus on the asymptotic trend of the system when the external input is held constant; such study becomes a matrix convergence problem by means of the transition matrices that define the system behavior. For the non-linear case a sufficient condition of convergence of the system (that, in particular, avoids oscillations) is demonstrated.


*Keywords:* Fuzzy feedback systems; Transition matrices; Standard additive model; Max–min; Sum-product

## 1. Introduction

Fuzzy feedback systems (FFSs) have been traditionally used as controllers, where they have demonstrated their effectiveness in a myriad of applications. The stability of these controllers is usually studied by non-linear analysis techniques, such as Lyapunov's Methods [10,11,15–18,21], which give a confidence of the reliability of the system, but such techniques do not characterize its recursive behavior.

There have been several interesting attempts to rigorously formalize FFSs; one of them is due to Tong [19] in 1980, where the author obtained a theoretical description of the closed loop response as a function of the initial state. Some interesting further works are the ones by Chen and Tsao [8] (where the authors stated that *the main cause of the failure of FFSs is due to the use of the max–min operator*) and Kang [12]. In the former, the key was to map the fuzzy system (FS) into a non-FS so that its behavior could be

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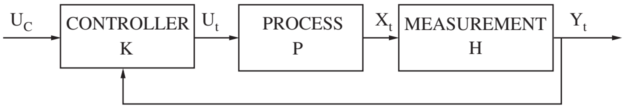


Fig. 1. Closed loop system [19].

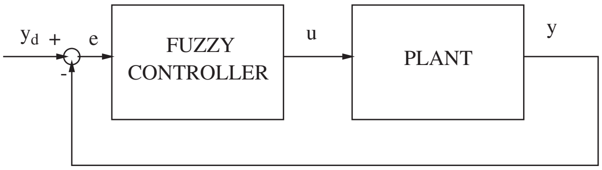


Fig. 2. Fuzzy control system [8].

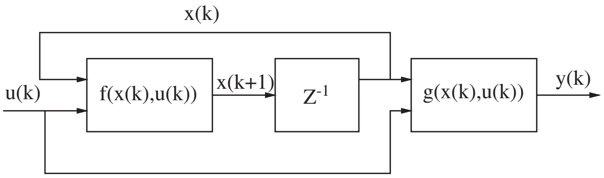


Fig. 3. Recurrent fuzzy systems [13,1].

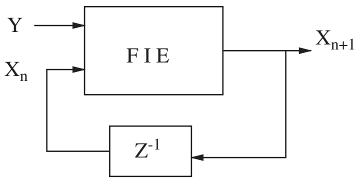


Fig. 4. Two-input one-output FFS.

better understood. In the latter, Kang proposed a systematic design method of linguistic fuzzy controllers. More recently, Adamy and Kempf [1,13] have introduced a particular case of FFS, the *recurrent FSs*, the study of which is based on the similarity with automata and recurrent neural networks. If the FFSs are appropriately designed they have an automaton-like behavior, but in other cases they may exhibit chaotic behavior. In a significant subclass of these systems the dynamic behavior can deduced directly from the rule base. Figs. 1–3 show the system configurations used in [19,8,13,1], respectively.

In this paper, we will focus on the analysis of FFSs where, unlike the cases depicted in Figs. 1 and 2, the (fuzzy) output is directly fed back into one of the inputs (see Fig. 4) without an intermediate defuzzification process. In this case, the output is a combination (linear or not) of the fuzzy sets defined over the output space, so the feedback input will also be. According to [3], a FS for which this requirement holds can be efficiently dealt with using transition matrices. By doing so, the input–output relation becomes a matrix relation and the analysis of the steady state of the system when the external input (the one not from the feedback) is held constant becomes a problem of matrix convergence.

Taking advantage of this property, in this paper, we will use the fast inference using transition matrices (FITM) methodology described in [3] to perform such a matrix analysis. This methodology has been