European Journal of Control (2008)4:286–296

DOI:10.3166/EJC.14.286–296

European Journal of Control

Modified Fast-sample/Fast-hold Approximation for Sampled-data System Analysis

Tomomichi Hagiwara*, Hiroaki Umeda**

Department of Electrical Engineering, Kyoto University, Kyotodaigaku-Katsura, Nishikyo-ku, Kyoto 615-8510, Japan

This paper deals with the H_∞ norm and frequency response gain analysis of sampled-data systems and provides a new approach, which we call modified fast-sample/fast-hold approximation. The new approximation approach discretizes the continuous-time generalized plant in a "γ-independent fashion" and leads to a discrete-time generalized plant with a similar structure to what is obtained by the conventional fast-sample/fast-hold approximation approach. Unlike the conventional approach, however, the modified fast-sample/fast-hold approximation approach can give both the upper and lower bounds of the H_∞-norm and/or the frequency response gain for sampled-data systems. Furthermore, the gap between the upper and lower bounds can be bounded from above directly from the fast-sample/fast-hold parameter N and is independent of the controller. These features are quite useful when it is applied to control system design, and this study indeed has very close relation with the control system design via *noncausal periodically time-varying scaling, a novel notion introduced recently.*

Keywords: Sampled-data systems, H_∞-norm, lifting, fast-lifting

1. Introduction

In the analysis and design of sampled-data systems, it is essential to deal with the intersample behavior of continuous-time signals as it is, and sophisticated techniques have been well established to that end, e.g., the lifting technique [3, 19–21], the FR-operator technique [2, 7], the approach with jump systems [12, 18] and the parametric transfer function approach [17]. These techniques can be regarded as providing methods for manipulating infinite-dimensional operators in the definitions of the H_∞-norm and/or frequency response gain of sampled-data systems [22] and then reducing the inherently infinite-dimensional analysis and design problems to finite-dimensional ones in an exact fashion.

An independent method for dealing with sampled-data systems, called fast-sample/fast-hold (FSFH) approximation [15, 23], is also well known on the other hand, which reduces the infinite-dimensional problems to finite-dimensional ones "in an asymptotically exact fashion" in the sense that the approximation error is ensured to converge to zero as the approximation parameter tends to ∞. The FSFH approximation, however, does not providea systematic way for determining some guaranteed upper and lower bounds of the H_∞-norm and/or frequency response gain of sampled-data systems for fixed N. This makes it difficult for the FSFH approximation approach to be applied to strict analysis and particularly control system design with guaranteed performance, essentially because we cannot determine when N is large enough to be able to ensure the associated analysis and design results exact and strict.

Motivated by our recent study on noncausal periodically time-varyingscaling of sampled-data systems [6, 10], we provide yet another method for the

---

*Correspondence to: T. Hagiwara, E-mail: hagiwara@kuee.kyoto-u.ac.jp
**E-mail: umeda@jaguar.kuee.kyoto-u.ac.jp

Received 6 August 2007; Accepted 3 June 2008
Recommended by S. Mondiè, A.J. van der Schaft

analysis of the H_∞-norm and/or frequency response gain of sampled-data systems in this paper. Regarding the H_∞ analysis, the new method has a feature that the associated discretization of the continuous-time generalized plant is carried out in a simple "γ-independent fashion" unlike in the exact methods [3, 11, 13, 14] that require so-called "γ-dependent discretization." Furthermore, the resulting discretized generalized plant has a structure that is quite similar to what is obtained by the conventional FSFH approximation [23]. While our new method is quite similar to the conventional FSFH approximation approach in these respects, ours has a distinctive advantage in that guaranteed upper and lower bounds of the H_∞-norm and frequency responsegain of sampled-data systems can be determined with it. Furthermore, a noteworthy fact regarding this advantage is that the gap between the upper and lower bounds can be bounded from above once we fix the continuous-time generalized plant and the approximation parameter N. This in particular implies that the gap is independent of the discrete-time controller. Hence, it follows that (i) we could judge whether or not the given approximation parameter N is large enough for the desired accuracy not only in analysis but also in design and (ii) the new method opens possibilities for sampled-data control system design with guaranteed robust stability or performance under plant uncertainties with some structures, unlike the conventional FSFH approximation. Taking all these advantages into consideration, we refer to our new method as modified fast-sample/fast-hold approximation.

This paper is organized as follows. We first review the lifted representation of sampled-data systems in Section 2, which is used throughout the paper. Then, in Section 3, we employ an operator of "fast-lifting", denoted by $\mathcal{L}_N$ and first introduced in [6], which plays a central role in this paper. Applying this operator, we then derive our main results about what we call modified FSFH approximation. Some remarks about the relationship with this study and noncausal periodically time-varying scaling [6, 10], as well as a relevant study giving another set of upper and lower bounds [9], are also given. Then, a numerical example is studied in Section 4 to verify the effectiveness of modified FSFH approximation. Finally, Section 5 concludes the paper.

2. Lifted Representation of Sampled-Data Systems

In this section, we review the lifted representation ofsampled-data systems [3, 21, 22], which will be used throughout the paper.

Let us consider the sampled-data system Σ shown in Fig. 1, where P denotes the continuous-time generalized plant, Ψ the discrete-time controller, $\mathcal{S}$ the ideal sampler and $\mathcal{H}$ the zero-order hold, both with sampling period h. Suppose that P and Ψ are described by the state-space representations

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= Ax + B_1 w + B_2 u, \\ z &= C_1 x + D_{12} u, \qquad y = C_2 x \end{aligned} \tag{1}$$

and

$$\psi_{k+1} = A_\Psi \psi_k + B_\Psi y_k, \quad u_k = C_\Psi \psi_k + D_\Psi y_k \tag{2}$$

respectively, where $y_k := y(kh)$ and $u(t) = u_k kh \le t < (k+1)h$). Let us denote by $\{\widehat{w}_k\}_{k=0}^\infty$ and $\{\widehat{z}_k\}_{k=0}^\infty$ the lifted representation of $w(t)$ and $z(t)$, respectively, with the sampling period h (i.e., $\widehat{w}_k(\theta) = w(kh+\theta)$ for $0 \le \theta < h$, and similarly for $\widehat{z}_k(\theta)$), and let $x_k := x(kh)$. Then, the lifted representation of P is given by

$$\begin{aligned} x_{k+1} &= A_d x_k + \mathbf{B}_1 \widehat{w}_k + B_{2d} u_k, \\ \widehat{z}_k &= \mathbf{C}_1 x_k + \mathbf{D}_{11} \widehat{w}_k + \mathbf{D}_{12} u_k, \qquad y_k = C_{2d} x_k \end{aligned} \tag{3}$$

with the matrices

$$A_d := \exp(Ah), \qquad B_{2d} := \int_0^h \exp(A\sigma) B_2 d\sigma,$$
$$C_{2d} := C_2 \tag{4}$$

and the operators

$$\mathbf{B}_1 : \mathcal{K} \ni w \mapsto \int_0^h \exp(A(h-\sigma)) B_1 w(\sigma) d\sigma \in \mathcal{F} \tag{5}$$

$$\mathbf{C}_1 : \ \mathcal{F} \ni x \mapsto z \in \mathcal{K}, \qquad z(\theta) = C_1 \exp(A\theta) x \tag{6}$$

$$\mathbf{D}_{11} : \mathcal{K} \ni w \mapsto z \in \mathcal{K}, \quad z(\theta) = \int_0^\theta C_1 \exp(A(\theta-\sigma)) B_1 w(\sigma) d\sigma \tag{7}$$

$$\mathbf{D}_{12} : \mathcal{F} \ni u \mapsto z \in \mathcal{K}, \quad z(\theta) = \int_0^\theta C_1 \exp(A(\theta-\sigma)) B_2 d\sigma\, u + D_{12} u \tag{8}$$

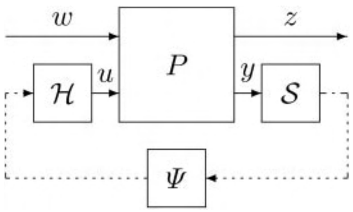

Fig. 1. Sampled-data system Σ.