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Duality in system analysis for bond graph models

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Abstract

Duality as a general notion has been discussed previously in multiple domains. The field of system modeling, analysis and control has used it for quite some time. The duality between the controllability and the observability is well known, especially in the case of the linear time-invariant systems. But when it comes to the linear systems in general, time-invariant or otherwise, the definitions become ambiguous. Even though there have been papers which use the state space representation or the module theoretical approach, a unified description has not been found yet. This paper is meant to fill in this gap, by using the bond graph representation. The bond graph perspective offers a global overview because the bond graph is a graphical tool which can be seen both as a state space representation and as a module.

Keywords: Duality; Bond graph; Linear time-varying models; Controllability; Observability; Dual bond graph model

1. Introduction

Usually a large range of procedures and algorithms for calculating best suited control laws are proposed to system control engineers. The techniques can be very different in their substance and the concepts can be understood only by skilled engineers or researchers. One goal is to link different kinds of model properties or control law properties. This link can be pointed out with the concept of duality.

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The concept of duality in the linear systems has been discussed from the state space representation point of view in [1,2] in the 1980s. In the late 1990s, the module theoretical approach offered a new perspective on the duality in linear systems [3].

The present paper brings a new perspective by using the bond graph, a graphical tool which can be seen at the same time as a state space representation and as a module. Therefore the procedures developed in both approaches can be applied on the bond graph models.

The interest of the article is in the duality in system analysis. The first part offers a recall of the duality from the two approaches, state space and module theoretical representations. The second part is focused on the definition of the dual bond graph model. In the third part the procedures used to prove the duality in the system analysis from the bond graph perspective are presented. In the final section, we have gathered the conclusions and some perspectives.

2. Algebraic framework. Properties of linear systems

The module theoretical approach introduced in [4], which defines a linear system model as a module, provides the mathematical framework needed in the sequel. First, a few basic information about rings and modules are recalled. Later on, a few insights in the module approach are presented in order to have a comprehensible description of the mathematical tool which is used.

2.1. Rings and modules

Let k be an arbitrary differential field, i.e. a commutative field equipped with a single derivation d/dt (in the sequel denoted also as s) such that, for all $a, b \in k$ properties (1) are verified:

$$\begin{cases} \dfrac{da}{dt} \in k \\ \dfrac{d}{dt}(a+b) = \dfrac{da}{dt} + \dfrac{db}{dt} \\ \dfrac{d}{dt}(ab) = \dfrac{da}{dt}b + a\dfrac{db}{dt} \end{cases} \tag{1}$$

For notational convenience $\dot{a}$ denotes da/dt. A constant is an element $c \in k$ such that $\dot{c} = 0$. A differential subfield of k, whose elements are constants, is called a field of constants. The field of constants is the one which characterizes the LTI systems.

The noncommutative ring $k[d/dt]$ of the polynomials of the form (2), denoted as R, is used in the general linear case, time-varying or time-invariant. Its elements can be understood as linear differential operators:

$$\sum_{finite} \alpha_i \frac{d^i}{dt^i}, \quad \alpha_i \in k \tag{2}$$

The multiplication in R is defined in [4] by

$$\frac{d}{dt}a = a\frac{d}{dt} + \dot{a}, \quad a \in k \tag{3}$$