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Design and performance analysis of adjustable window functions based cosine modulated filter banks

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ABSTRACT

In this paper, design and comparative analysis of adjustable window functions based cosine modulated filter banks are analyzed. Four adjustable windows, viz., Kaiser window, Saramäki window, ultraspherical windows and Roark's transitional window are used to design prototype filters. Reconstruction error, which is used as an objective function, is minimized by optimizing the cutoff frequency of designed prototype filters. The gradient based iterative optimization algorithm is used. These optimized filters are later cosine modulated to obtain filter banks. The performances of filter banks are compared on the basis of reconstruction error and aliasing error.

1. Introduction

The multirate filter banks find variety of applications in subband coding, transmultiplexing, image and video or audio compression, spectral estimation, biosignal processing, and adaptive signal processing. The elementary block in realization of such applications is cosine modulated filter banks (CMFBs), in which analysis and synthesis filter banks are obtained by cosine modulated versions of lowpass prototype filter [1–3], Fig. 1. This scheme is popular for its ease of designing high-selective and high-discrimination systems. Also, since analysis and synthesis filter banks are generated by the lowpass prototype filter, the entire design of the filter bank is reduced to the design of the prototype filter. Therefore, during the design phase, it is required to optimize the coefficients of the prototype filter only. This significantly reduces the complexities and computational overheads.

Suppose the prototype filter $P(e^{j\omega})$ is a low pass linear phase. The quality of reconstructed signal depends on how closely $P(e^{j\omega})$ satisfies the following two conditions:

$$\left|P\left(e^{j\omega}\right)\right| = 0 \quad \text{for } |\omega| > \pi/M \tag{1}$$

and

$$\left|T\left(e^{j\omega}\right)\right| = 1 \quad \text{for } 0 < \omega < \pi/M$$

$$\text{where } T\left(e^{j\omega}\right) = \sum_{k=0}^{2M-1} \left|P\left(e^{j(\omega - k\pi/M)}\right)\right|^2 \tag{2}$$

where M is the number of channels.

If (1) is satisfied exactly, there is no aliasing between nonadjacent bands, while if (2) is satisfied exactly, amplitude distortion is completely eliminated in the combined analysis/synthesis filter bank system. Aliasing between the adjacent bands is eliminated by selecting appropriate phase factor in the modulation [3]. Unfortunately, it is not possible to design a finite length filter that exactly satisfies the constraints of (1) and (2). Hence the filter bank that provides approximate or near-perfect reconstruction (NPR) is designed that approximately satisfies the constraints laid down in (1) and (2). Different objective functions have been optimized using linear optimization [4,5] as well as nonlinear optimization techniques [6,7].

2. Filter bank design

The analysis and synthesis filter banks are based on cosine modulation of $P(e^{j\omega})$. The prototype filter with desired characteristics can be easily designed by window technique.

2.1. Window technique

The impulse response of the ideal low pass filter with cutoff frequency ω_c is given as

$$h_{id}(n) = \frac{\sin \omega_c n}{\pi n}, \quad -\infty < n < \infty \tag{3}$$

$h_{id}(n)$ extends from $-\infty$ to $+\infty$, is not absolutely summable and, therefore, unrealizable [8]. Hence, shifted impulse response of $h_{id}(n)$ will be

$$h_{id}(n) = \frac{\sin(\omega_c(n - 0.5N))}{\pi(n - 0.5N)}, \quad n \in Z \tag{4}$$

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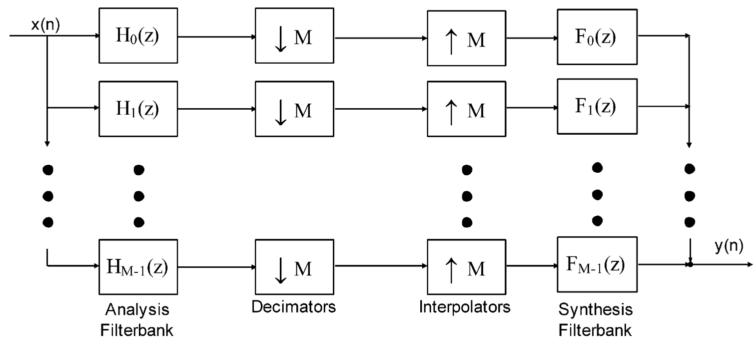

Fig. 1. M-channel maximally decimated filter bank.

For making a causal filter, direct truncation of infinite-duration impulse response of a filter results in large passband and stopband ripples near transition band. These undesired effects are well known Gibbs' phenomenon. However these effects can be significantly reduced by appropriate choice of smoothing function $w(n)$. Hence, a filter $p(n)$ of order N is of the form [9]:

$$p(n) = h_{id}(n)w(n) \tag{5}$$

where $w(n)$ are the time domain weighting functions or window functions. Window function is of limited duration in time domain, which approximates band limited function in frequency domain. Window functions are broadly categorized as fixed and adjustable windows. In fixed window, the window length N governs mainlobe width. Adjustable window has two or more independent parameters that control the window's frequency response characteristics.

2.2. Adjustable window functions

2.2.1. Kaiser window

Kaiser window [10,11] achieves close approximations to the discrete prolate functions, which have a maximum energy concentration in the main lobe relative to that of the side lobes.

The window function $w(n)$ for Kaiser window is given as [8]:

$$w(n) = \begin{cases} \frac{I_0\{\beta\sqrt{1-(n/N)^2}\}}{I_0(\beta)}; & 0 \leqslant n \leqslant N \\ 0; & \text{otherwise} \end{cases} \tag{6}$$

where $I_0(.)$ is the zeroth-order modified Bessel function, which can be computed as:

$$I_0(x) = 1 + \sum_{k=1}^{\infty}\left(\frac{(0.5x)^k}{k!}\right)^2 \tag{7}$$

Parameter β for desired A_s and filter order N, an appropriate chosen transition bandwidth $\Delta\omega$, can be estimated as:

$$\beta = \begin{cases} 0.1102(A_s - 8.7); & A_s > 50 \\ 0.5842(A_s - 21)^{0.4} + 0.07886(A_s - 21); & 21 \leqslant A_s \leqslant 50 \\ 0; & A_s < 21 \end{cases} \tag{8}$$

$$N \approx \frac{A_s - 7.95}{14.36\Delta\omega/2\pi} \tag{9}$$

2.2.2. Saramäki window

This window [12,13] is also close approximation to discrete prolate functions. Compared to Kaiser window, it has the advantage of having analytical expression in both the frequency and time domains. Further, no power series expressions are needed in evaluating the window functions. The FIR filters obtained are slightly better than those obtained using Kaiser window. The window function $w(n)$ for the window is given as:

$$w(n) = v_0(n) + 2\sum_{k=1}^{N} v_k(n) \tag{10}$$

and

$$N = \frac{A_s - 8.15}{14.36(\omega_s - \omega_p)/\pi} \tag{11}$$

$v_k(n)$ can be calculated according to the following recursion relations:

$$v_0(n) = \begin{cases} 1; & n = 0 \\ 0; & \text{otherwise} \end{cases} \tag{12a}$$

$$v_1(n) = \begin{cases} \gamma - 1; & n = 0 \\ \gamma/2; & |n| = 1 \\ 0; & \text{otherwise} \end{cases} \tag{12b}$$

and

$$v_k(n) = \begin{cases} 2(\gamma - 1)v_{k-1}(n) - v_{k-2}(n) + \gamma[v_{k-1}(n-1) \\ \quad + v_{k-1}(n+1)]; & |n| \leqslant k \\ 0; & \text{otherwise} \end{cases} \tag{12c}$$

Parameter γ can be computed as

$$\gamma = \frac{1 + \cos\frac{2\pi}{2N+1}}{1 + \cos\frac{2\beta\pi}{2N+1}} \tag{13}$$

with

$$\beta = \begin{cases} 0.000121(A_s - 21)^2 + 0.0224(A_s - 21) \\ \quad + 1; & 21 \leqslant A_s \leqslant 65 \\ 0.033A_s + 0.062; & 65 < A_s \leqslant 110 \\ 0.0342A_s - 0.064; & A_s > 110 \end{cases} \tag{14}$$

2.2.3. Ultraspherical window

These windows are based on orthogonal polynomial known as Gegenbauer or ultraspherical polynomial. These polynomials have a close relationship with Jacobi polynomial and Chebyshev polynomial. The window has three control parameters with additional capability of generating variety of sidelobe patterns.

The coefficients of ultraspherical windows can be generated as:

$$w(nT) = \frac{1}{N}\left[C_{N-1}^{\alpha}(x_0) + \sum_{i=1}^{(N-1)/2} C_{N-1}^{\alpha}\left(x_0\cos\frac{i\pi}{N}\right)\cos\left(\frac{2n\pi i}{N}\right)\right] \tag{15}$$

where $C_N^{\alpha}(x)$ is the ultraspherical polynomial of degree N defined by recursion relationship