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Optimal linear spatial filters for event-related potentials based on a spatio-temporal model: Asymptotical performance analysis

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ABSTRACT

In this paper, the estimation of spatio-temporal patterns in the context of event-related potentials or evoked potentials studies in neuroscience is addressed. The proposed framework (denoted xDAWN) has the advantage to require only the knowledge of the time of stimuli onsets which are determined by the experimental setup. A theoretical analysis of the xDAWN framework shows that it provides asymptotically optimal spatial filters under weak assumptions. The loss in signal to interference-plus-noise ratio due to finite sample effect is calculated in a closed form at the first order of perturbation and is then validated by simulations. This last result shows that the proposed method provides interesting performance and outperforms classical methods, such as independent component analysis, in a wide range of situations. Moreover, the xDAWN algorithm has the property to be robust with respect to the model parameter values. Finally, validations on real electro-encephalographic data confirm the good behavior of the proposed xDAWN framework in the context of a P300 speller brain–computer interface.

1. Introduction

In cognitive neuroscience, it is useful to explore brain activity through evoked potentials (EPs) or event-related potentials (ERPs) recorded by electro-encephalography (EEG), e.g. [1,2]. For instance, ERPs allow to investigate (i) the basic functional pathways through early ERPs or EPs as auditory, visual or somatosensory networks, and (ii) cognitive pathways through late ERPs which are more related to memory tasks, execution of attention and emotion. ERP experiments usually involve the presentation of several kinds of stimuli and suppose that there exists a typical spatio-temporal pattern which is time-locked to each kind of stimuli (also called events).

In this context, EEG recorded signals do not only contain the spatio-temporal patterns linked to the events but also ongoing brain activity as well as muscular and/or ocular artifacts. As a consequence, to ease the estimation of such spatio-temporal patterns, one can repeat the experiments but this solution needs to record more data. This method is based on the assumption that the ERP waveforms are uncorrelated with the ongoing cerebral activity and with the artifacts: the ERP waveforms can thus be estimated by a straightforward or a weighted average of the trials temporally aligned to the stimuli onsets [3]. The main drawback of this approach is that it only exploits the temporal aspect of the ERP. Another typical way to improve these estimates is to enhance the ERPs by a spatial filtering of the channels. Several methods based on independent component analysis (ICA) [4–8] have thus been proposed to enhance the signal-to-noise ratio (SNR) or to remove the artifacts, e.g., [9–11]. In addition, after the optimization stage, these methods need

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to select the components (manually or using spatio-temporal prior knowledge). However, these methods often fail to extract correctly the ERP component since in a real experiment, the ERP components have a very small amplitude (about μV) compared to ongoing cerebral activity (about mV) and to ocular artifacts (about 100 mV). These methods are mainly based on spatial assumptions and do not exploit the temporal structures of the ERPs.

To avoid such limitations, methods based on a spatio-temporal model have been developed. For instance, common spatial pattern (CSP) [12,13] or Fisher's linear discriminant analysis (LDA) [14] are two classical methods to estimate spatial filters. CSP aims at simultaneously maximizing the power of one ERP and minimizing the power the other ERPs: it tries to maximize the signal-to-interference ratio (SIR). LDA is based on the maximization of the distance between two classes while it minimizes the variance within each class. More recently, several methods (e.g., [15–17]) investigate more complex spatio-temporal models. For instance in [16], a regular parametric waveform of the ERP is imposed to estimate the spatial filters. In [17], a direct estimation of the temporal waveform and the related spatial distribution without parameter selection has been proposed. However, all these methods are not able to deal with ERP waveforms that can temporally overlap each others with correlation, within one kind of ERPs and/or between several kinds of ERPs. In our previous studies [18,19], the xDAWN algorithm has been introduced. It aims at estimating jointly the temporal signature and the spatial distribution of the ERPs as well as the spatial filters that provide the largest signal-to-signal-plus-noise ratio (SSNR). The main advantage of this framework is its absence of assumptions either on the temporal waveform and the spatial distribution. The only prior knowledge is the onsets of the stimuli used in the experiment. In this contribution, a theoretical analysis of xDAWN framework is derived: it shows that the proposed method (i) is asymptotically optimal and (ii) has a good behavior, at the first order of perturbations, by substituting exact parameter values by estimated ones from the data. In addition, since no particular assumptions are imposed, the proposed xDAWN framework can be easily adopted for solving similar estimation problems if the proposed model is verified.

The rest of this paper is organized as follows. Section 2 summarizes the xDAWN framework. The theoretical analysis of its optimality and the asymptotical performance analysis are derived in Section 3. Section 4 investigates the links between xDAWN algorithm and other classical methods to estimate spatial filters in an ERP paradigm. Section 5 presents numerical experiments and validation on real EEG data, and Section 6 concludes this paper.

2. xDAWN spatial filters

In this section, the proposed xDAWN framework is briefly summarized.

2.1. Model

In the context of ERPs analysis, which supposes that there exists a typical spatio-temporal pattern time-locked with the stimuli, EEG signals $\mathbf{x}(k) \in \mathbb{R}^{N_s}$ recorded from N_s sensors can be modeled as the superposition of the N_e signals related to each of the N_e classes of events (i.e. kinds of stimulations) and ongoing brain activity as well as ocular and/or muscular artifacts $\mathbf{n}(k) \in \mathbb{R}^{N_s}$. To take into account the variability of each ERP in a particular class that can appear during the experiment, one can assume that the j-th ERP of the i-th class, denoted $\mathbf{p}_{i,j}(k) \in \mathbb{R}^{N_s}$, is composed of a spatio-temporal pattern, $\mathbf{p}_i^{(c)}(k) \in \mathbb{R}^{N_s}$, common to all ERPs of the i-th class and of a random spatio-temporal pattern $\mathbf{p}_{i,j}^{(r)}(k) \in \mathbb{R}^{N_s}$ different for all ERPs of the i-th class:

$$\mathbf{p}_{i,j}(k) = \mathbf{p}_i^{(c)}(k) + \mathbf{p}_{i,j}^{(r)}(k).$$

As a consequence, one can model the raw EEG as

$$\mathbf{x}(k) = \sum_{i=1}^{N_e} \sum_{j=1}^{K_i} \mathbf{p}_{i,j}(k - \tau_i(j)) + \mathbf{n}(k), \tag{1}$$

where $\tau_i(j)$ is the index time of the j-th stimulus of the i-th ERP class and K_i is the number of stimuli of the i-th ERP class. Basic algebraic manipulations lead to rewrite the convolutional model (1) in matrix notation as

$$X = \sum_{i=1}^{N_e} \sum_{j=1}^{K_i} D_{i,j} P_{i,j} + N, \tag{2}$$

where the k-th row of $X \in \mathbb{R}^{N_t \times N_s}$ (resp. N) is $\mathbf{x}(k)^T$ (resp. $\mathbf{n}(k)^T$) and N_t is the total number of time samples. $\cdot^T$ is the transpose operator. $P_{i,j} \in \mathbb{R}^{M_i \times N_s}$ is the j-th ERP spatio-temporal pattern of the i-th class of stimuli whose k-th row is $\mathbf{p}_{i,j}(k)^T$. $D_{i,j} \in \mathbb{R}^{N_t \times M_i}$ is a Toeplitz matrix whose first column entries are null but $D_{i,j}(\tau_i(j),1) = 1$. M_i is the number of time samples of the temporal pattern of i-th class of ERPs. In (2), $\sum_j D_{i,j} P_{i,j}$ thus models the signals related to the i-th class of events. Since $P_{i,j}$ is often a singular matrix (i.e. of reduced rank), spatio-temporal patterns can be factorized as $P_{i,j} = A_{i,j} W_{i,j}^T$, where $A_{i,j} \in \mathbb{R}^{M_i \times N_{s_i}}$ is temporal pattern of reduced dimensions and $W_{i,j} \in \mathbb{R}^{N_s \times N_{s_i}}$ is its spatial distribution over sensors, with $N_{s_i} < N_s$.

Moreover, one can assume that the differences between spatio-temporal patterns $P_{i,j}$ among the same class of ERP only come from temporal differences and not from spatial ones[1]:

$$P_{i,j} = \left(A_i^{(c)} + A_{i,j}^{(r)}\right) W_i^T,$$

where $A_i^{(c)} \in \mathbb{R}^{M_i \times N_{s_i}}$ denotes the common temporal pattern and $A_{i,j}^{(r)} \in \mathbb{R}^{M_i \times N_{s_i}}$ models the random temporal pattern. As a consequence, model (2) can be expressed as

$$X = \sum_{i=1}^{N_e} \left(D_i^{(c)} A_i^{(c)} + D_i^{(r)} A_i^{(r)}\right) W_i^T + N, \tag{3}$$

where $D_i^{(c)} = \sum_{j=1}^{K_i} D_{i,j}$, $D_i^{(r)} = [D_{i,1}, \ldots, D_{i,K_i}]$ and $A_i^{(r)} = [A_{i,1}^{(r)T}, \ldots, A_{i,K_i}^{(r)\,T}]^T \in \mathbb{R}^{(M_i K_i) \times N_{s_i}}$ are white centered Gaussian random variables.

In ERP analysis, one is generally only interested in the common (averaged) temporal patterns $A_i^{(c)}$.

[1] This is a reasonable assumption, since one can assume that the neurons involved in a specific cognitive task remain the same during the experiment while their temporal activity could be different.