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Performance analysis of parallel Schwarz preconditioners in the LES of turbulent channel flows[☆]

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ABSTRACT

We present a comparative study of parallel Schwarz preconditioners in the solution of linear systems arising in a Large Eddy Simulation (LES) procedure for turbulent plane channel flows. This procedure applies a time-splitting technique to suitably filtered Navier–Stokes equations, in order to decouple the continuity and momentum equations, and uses a semi-implicit scheme for time integration and finite volumes for space discretisation. This approach requires the solution of four sparse linear systems at each time step, accounting for a large part of the overall simulation; hence the linear system solvers are a crucial component in the whole procedure. Several preconditioners are applied in the simulation of a reference test case for the LES community, using discretisation grids of different sizes, with the aim of analysing the effects of different algorithmic choices defining the preconditioners, and identifying the most effective ones for the selected problem. The preconditioners, coupled with the GMRES method, are run within SParC-LES, a recently developed LES code based on the PSBLAS and MLD2P4 libraries for parallel sparse matrix computations and preconditioning.

1. Introduction

Large Eddy Simulation (LES) is a widely used approach for detailed study of turbulent flows in small/medium-scale applications. Although it has a lower computational cost than Direct Numerical Simulation (DNS), its application to realistic flows remains a computationally intensive procedure. In this work we focus on large and sparse linear systems arising in a LES procedure for turbulent wall-bounded flows, and analyse the performance of parallel Schwarz preconditioners in solving these systems by GMRES [1]. The linear systems stem from the application of a time-splitting technique to suitably filtered Navier–Stokes equations, to decouple the continuity and momentum equations, and from a finite-volume discretisation of the resulting equations (see the next section for details). Their solution accounts for a significant part of the overall computational effort; thus the use of efficient preconditioners is critical for the efficiency of the overall simulation.

In our analysis we use the preconditioners implemented in the Multilevel Domain Decomposition Parallel Preconditioners Package based on PSBLAS (MLD2P4) [2], coupled with the GMRES solver from the Parallel Sparse BLAS (PSBLAS) library [3]. The solution of the systems is performed within SParC-LES [4], a parallel code recently developed for the simulation of turbulent channel flows, which is run on a test case used as a standard benchmark in the Italian

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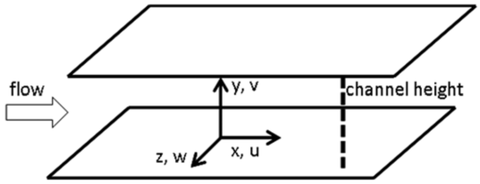

Fig. 1. Channel geometry.

LES community [5]. This study differs from previous work in [6,7] because it provides a more detailed analysis of several preconditioners for all the linear systems arising in the LES procedure, applied to a widely used reference test case. It is also worth noting that the results discussed in this paper guided our choice of the preconditioners in a complete simulation of the selected test case with the SParC-LES code (see [4]).

The paper is organised as follows. In Section 2 we briefly outline the LES approach implemented in SParC-LES, with the aim of introducing the above-mentioned linear systems. In Section 3 we give a very brief overview of the preconditioners implemented in MLD2P4. Finally, in Section 4 we present our analysis of the preconditioners. We also provide some conclusions in Section 5.

2. Sparse linear systems in the LES of turbulent channel flows

We consider the approach proposed in [8,9] for simulating an incompressible and homothermal flow in a plane channel. The LES governing equations are obtained by applying a top-hat filter coupled with a differential deconvolution operator to the Navier–Stokes (N–S) equations in non-dimensional weak conservation form. Periodic boundary conditions are prescribed in the streamwise (x) and spanwise (z) directions, and no-slip conditions on the walls (y). A picture showing the channel geometry and the orientation of the coordinate directions is provided in Fig. 1. The numerical solution of the filtered N–S equations is based on a time-splitting technique, using an approximate projection method, to decouple the continuity equation from the momentum equation. Thus, at the n-th time step, the unknown velocity field $\widetilde{\mathbf{v}}^n$ is computed through the following predictor–corrector formula:

$$\widetilde{\mathbf{v}}^n = \mathbf{v}^* - \Delta t \nabla \phi, \tag{1}$$

where $\mathbf{v}^*$ is an intermediate velocity field, Δt is the time step size, and ϕ is a suitable scalar field such that $\widetilde{\mathbf{v}}^n$ is divergence-free.

The intermediate velocity $\mathbf{v}^*$ is computed by solving a modified momentum equation, where the pressure term is neglected, with Dirichlet boundary conditions at the walls. By appling a second-order Adams–Bashforth/Crank–Nicolson semi-implicit scheme, the following equation is obtained:

$$\begin{aligned}\left(A_{\mathbf{x}}^{-1} - \frac{\Delta t}{2\mathrm{Re}} D_2\right)\mathbf{v}^* &= \left(A_{\mathbf{x}}^{-1} + \frac{\Delta t}{2\mathrm{Re}} D_2\right)\widetilde{\mathbf{v}}^{n-1} \\ &\quad + \frac{\Delta t}{2}\left(3\left(\frac{1}{\mathrm{Re}}(D_1+D_3)\widetilde{\mathbf{v}}^{n-1} + \mathbf{f}_{\mathrm{conv}}^{n-1}\right) - \left(\frac{1}{\mathrm{Re}}(D_1+D_3)\widetilde{\mathbf{v}}^{n-2} + \mathbf{f}_{\mathrm{conv}}^{n-2}\right)\right),\end{aligned} \tag{2}$$

where $A_{\mathbf{x}}$ is the differential deconvolution operator, D_1, D_2 and D_3 are suitable diffusion operators, $\mathbf{f}_{\mathrm{conv}}$ is the so-called convective flux, Re is the Reynolds number, and the superscripts $n-1$ and $n-2$ indicate that a quantity is computed at the $(n-1)$-th and $(n-2)$-th time steps, respectively.

The scalar field ϕ is obtained by solving the following Poisson-like equation:

$$(D_1 + D_2 + D_3)\phi = \frac{1}{\Delta t\, |\Omega(\mathbf{x})|} \int_{\partial\Omega(\mathbf{x})} \mathbf{v}^* \cdot \mathbf{n}\, dS, \tag{3}$$

where $\Omega(\mathbf{x})$ is a finite volume contained within the flow region and $\mathbf{n}$ is the outward-oriented unit vector normal to $\partial\Omega(\mathbf{x})$. Neumann boundary conditions are prescribed in the wall-normal direction, which satisfy the compatibility conditions and hence ensure the existence of a solution that is unique up to an additive constant. Note that (3) is also known as the pressure equation, since $\nabla\phi$ provides an $O(\Delta t)$ approximation of the pressure gradient.

Eqs. (2) and (3) are discretised in space by using a structured Cartesian grid, with uniform spacings in the streamwise and spanwise directions, where the flow is assumed to be homogeneous, and a non-uniform grid spacing with refinement near the walls in the y direction, to capture the fluid behaviour at the boundary layer. The equations are discretised by using a finite volume method, with the flow variables $\mathbf{v}^*$ and ϕ co-located at the centres of the control volumes.

In Eq. (2), a third-order multidimensional upwind scheme is used for the convective fluxes, and a classical second-order central scheme for the diffusive fluxes; a fourth-order central scheme is applied for the discretisation of the inverse deconvolution operator. The discrete deconvolved momentum equation consists of three linear systems:

$$A_v v_r^* = w_r, \quad r = 1, 2, 3, \tag{4}$$