Systems & Control Letters 61 (2012) 549–554

Contents lists available at SciVerse ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Passivity of switched linear systems: Analysis and control design

José C. Geromel [a], Patrizio Colaneri [b], Paolo Bolzern [b,*]

[a] School of Electrical and Computer Engineering, UNICAMP, 13083 - 852, Campinas, SP, Brazil

[b] Politecnico di Milano, Dipartimento di Elettronica e Informazione, Piazza Leonardo da Vinci 32, 20133 Milano, Italy

ARTICLE INFO

Article history:
Received 12 April 2011
Received in revised form
9 November 2011
Accepted 15 February 2012
Available online 22 March 2012

Keywords:
Passivity
Switched systems
Dwell-time
LMI conditions

ABSTRACT

A passive system with positive definite storage function is not only stable but is intrinsically robustly stable with respect to a wide class of feedback disturbances. For linear time invariant systems, passivity can be characterized either in time domain or in frequency domain from positive realness. This paper aims to generalize this concept to continuous-time switched linear systems. Analysis is performed by taking into account state dependent and arbitrary time dependent switching functions with a prescribed dwell time. A control design problem related to the determination of a switching strategy, based upon output measurements, that renders a switched linear system passive is also considered. The methods introduced in the paper can be effectively applied to the control of the duty cycle and passivation of switched circuits.

1. Introduction

Recently, much attention has been paid in the circuit and control communities to switched dynamical systems, where the continuous-time dynamics is subject to abrupt changes according to a switching signal, either exogenous or controlled. Switched circuits and power converters are natural examples of interesting applications of switched systems theory. The stability analysis of continuous-time switched linear systems has been addressed by many authors, [1–6]. General results on this topic are presented in the book [7] and in the survey papers [8,9]. The reader is requested to see [10,11] for complete reviews on stability of continuous-time switched linear systems.

Passivity is one of the main concepts at the intersection of circuit and control theory, [12,13]. Roughly speaking, a system is passive if it dissipates energy without generating its own. This notion encompasses the property of stability, and positive storage functions that characterize passivity can be qualified as Lyapunov functions. In this framework, passivity is intimately related to stability robustness of the system under negative feedback perturbations belonging to certain classes, including sector static nonlinearities and passive dynamical systems, [14].

The passivity concept of general nonlinear switched systems was originally considered in [15] where multiple positive definite storage functions have been successfully used for stability analysis and for the design of a state dependent switching law that renders the closed-loop system passive, see also [16]. In switched circuits, the passivity property proved to be a powerful tool for the control of power converters, [17,18], and fault tolerant control of RLC circuits, [19].

The present paper is focused on the class of switched linear systems, whose passivity properties are investigated by resorting to suitable linear matrix inequality (LMI) conditions, similarly to what is usually done for linear time-invariant systems (LTI). Differently from the latter case, our analysis calls for piecewise quadratic positive storage functions. The paper can be seen as an extension of the conference paper [20].

Considering a switched linear system with input $w(\cdot) \in \mathbb{R}^m$ and output $z(\cdot) \in \mathbb{R}^m$, the main goal is to design a switching signal such that the closed-loop system satisfies

$$\int_0^\infty z(t)'w(t)\mathrm{d}t \geq 0, \quad \forall w \in \mathcal{L}_2. \tag{1}$$

To this end two classes of design problems are considered. The first one, called *exogenous switching*, is characterized by switching signals $\sigma(\cdot)$ with a given dwell time $T > 0$, that is $\sigma \in \mathcal{D}_T$. An upper bound of the minimum dwell time T_* is determined preserving the validity of (1) for all $\sigma \in \mathcal{D}_{T_*}$. The complete solution to this problem is extremely difficult to obtain and, to our best knowledge, only few references are available up to now in the literature, see [15,16] and the related papers on RMS gain [21–23]. The design problem, converted into an optimal control problem is characterized through the Hamilton–Jacobi-Bellman equation which is virtually impossible to solve due to the algebraic structure of the set $\mathcal{D}_T$ for $T > 0$ given. Refs. [24,25] give an idea of the

* Corresponding author.
E-mail addresses: geromel@dsce.fee.unicamp.br (J.C. Geromel), colaneri@elet.polimi.it (P. Colaneri), bolzern@elet.polimi.it (P. Bolzern).

0167-6911/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.sysconle.2012.02.008

difficulties if one want to solve this kind of optimal control problem. Hence, for the moment, suboptimal solutions easier to calculate are acceptable. Notice that the knowledge of a dwell time guaranteeing passivity can be useful in the design of the duty-cycle of switched circuits.

The second class, called *controlled switching*, is characterized by assuming that the switching signal is of the form $\sigma(t) = u(x(t))$ where $u(\cdot): \mathbb{R}^n \to \mathbb{K} = \{1, 2, \ldots, N\}$ is a state-dependent switching function to be determined in order to preserve the inequality (1). As an important generalization we analyze the possibility to define the switching function using only partial information provided by a measured output. This control strategy can be beneficial for passivation via switching of nonpassive circuits.

The paper is organized as follows. After some preliminaries presented in Section 2, the main results for LTI systems concerning passivity are recalled in Section 3. The dwell-time analysis problem with exogenous switchings is treated in Section 4 whereas the controlled switching design is dealt with in Section 5. The paper ends with Section 6 where two simple examples are discussed. The first concerns the computation of the dwell-time preserving passivity of a switched system composed by two passive subsystems. The second puts in evidence the usefulness of the design results in providing switching strategies that are able to orchestrate two LTI non passive subsystems to become a passive switched linear system.

The notation used throughout is standard. For real matrices or vectors ($'$) indicates the transpose. The symbol ($\bullet$) denotes the symmetric blocks of a symmetric matrix. The set $\mathscr{M}_c$ denotes the set of Metzler matrices $\Pi \in \mathbb{R}^{N\times N}$ with nonnegative off diagonal elements satisfying the normalization constraints $\sum_{j=1}^{N} \pi_{ji} = 0$ for all $i = 1, \ldots, N$. Finally, the square norm of a trajectory $s(t)$ defined for all $t \geq 0$, denoted by $\|s\|_2^2$, equals $\|s\|_2^2 = \int_0^\infty s(t)' s(t)\mathrm{d}t$. All trajectories with bounded norm constitute the set $\mathscr{L}_2$. The set of all nonnegative integers is denoted by $\mathbb{N}$.

2. Preliminaries and problem statement

Consider a continuous-time linear switched system

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}w(t) \quad (2)$$

$$y(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}w(t) \quad (3)$$

$$z(t) = E_{\sigma(t)}x(t) + F_{\sigma(t)}w(t) \quad (4)$$

with $x(0) = 0$. The vectors $x(t) \in \mathbb{R}^n$, $w(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$ and $z(t) \in \mathbb{R}^m$ denote the state, the exogenous disturbance assumed to be in $\mathscr{L}_2$, the measured output and the controlled output variables, respectively. The disturbance input and the controlled output vectors have the same dimensions. The switching signal $\sigma(\cdot)$ selects at each instant of time, one and only one among N known linear subsystems (also called modes), that is $\sigma: \mathbb{R}_+ \to \mathbb{K}$. For a given switching signal, the following cost is considered

$$J(\sigma) = \sup_{w\in\mathscr{L}_2} -\int_0^\infty z(t)'w(t)\mathrm{d}t \quad (5)$$

which allows the conclusion that whenever $\sigma(t) = i \in \mathbb{K}$ is constant for all $t \geq 0$, then $J(\sigma) = 0$ if and only if the i-th subsystem is stable and passive or, equivalently, if it has a positive real transfer function. Two distinct classes of switching functions are of interest, namely:

- *exogenous switching*—The switching signal $\sigma(\cdot)$ is piecewise constant and satisfies a given dwell time $T > 0$. The set of all functions with these properties is defined as $\mathscr{D}_T = \{\sigma(\cdot): t_{k+1} - t_k \geq T, \forall k \in \mathbb{N}\}$ where t_{k+1} and t_k denote successive switching times. The extreme values of T in the time interval $(0, +\infty)$ correspond to arbitrary or constant switching signals, respectively.
- *controlled switching*—The switching function is of the form $\sigma(t) = u(y(t))$ where the output dependent function $u(\cdot)$ is to be designed. The set of all such functions that assure global asymptotic stability of the closed-loop switched system is denoted by $\mathscr{S}$. Pure state feedback can be considered by setting $C_i = I$ and $D_i = 0$ for all $i \in \mathbb{K}$.

Of course, each class induces a different design problem. For exogenous switching the goal is to determine

$$\gamma(T) = \sup_{\sigma\in\mathscr{D}_T} J(\sigma) \quad (6)$$

which characterizes the worst case performance with respect to the choice of σ exhibiting a prescribed dwell time $T > 0$. The main interest is on the determination of the smallest dwell time $T_* > 0$ such that $\gamma(T_*) = 0$, which can be alternatively calculated from

$$T_* = \inf_{T>0}\{T: J(\sigma) \leq 0, \forall\sigma \in \mathscr{D}_T\}. \quad (7)$$

Since the set $\mathscr{D}_T$ includes all constant signals $\sigma(t) = i, \forall i \in \mathbb{K}$, the problem makes sense only if each subsystem is passive, that is $J(\sigma) = 0$ for all $\sigma \in \mathscr{D}_\infty$.

On the other hand, for the class of controlled switching, the problem to be solved can be stated as

$$\mu = \inf_{\sigma\in\mathscr{S}} J(\sigma) \quad (8)$$

and the goal is to determine conditions assuring that $\mu = 0$. Since whenever at least one subsystem is passive we obtain $\mu \leq 0$, the challenge is to get $\mu = 0$ as the optimal solution of (8) even though all subsystems are not passive. From now on, it is important to keep clear that the optimal solution of each proposed problem will not be exactly calculated. Actually, only suboptimal solutions are worked out.

3. LTI systems passivity

In this section, the following LTI system with input and output vectors of the same dimensions is considered

$$\dot{x}(t) = Ax(t) + Bw(t) \quad (9)$$

$$z(t) = Ex(t) + Fw(t). \quad (10)$$

Assuming that it evolves from a zero initial condition, passivity requires that (see [26, p. 93])

$$\int_0^\infty z(t)'w(t)\mathrm{d}t \geq 0, \quad \forall w \in \mathscr{L}_2 \quad (11)$$

which is enforced by the quadratic storage function $v(x) = x'Px$, with $P > 0$, satisfying

$$\dot{v}(x) < z'w + w'z, \quad \forall(x, w) \neq 0. \quad (12)$$

From $w = 0 \in \mathscr{L}_2$ we notice that (12) implies asymptotic stability. Hence, using the system Eqs. (9)–(10) this inequality provides the (strictly) passivity condition, expressed in terms of the linear matrix inequalities, [12]

$$\begin{bmatrix} A'P + PA & PB - E' \\ \bullet & -F - F' \end{bmatrix} < 0, \quad P > 0. \quad (13)$$

Likewise, under mild assumptions involving controllability of the pair (A, B) and observability of (A, E), passivity can be alternatively characterized by

$$J = \sup_{w\in\mathscr{L}_2} -\int_0^\infty \left(z(t)'w(t) + w(t)'z(t)\right)\mathrm{d}t \quad (14)$$

subject to the system Eqs. (9)–(10) with the initial condition $x(0) = x_0$. Assuming $F + F' > 0$, the Hamiltonian is concave which assures the existence and unicity of the optimal solution. It provides