Physica A 391 (2012) 4827–4838

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Modeling stock price dynamics by continuum percolation system and relevant complex systems analysis

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A R T I C L E   I N F O

Article history:
Received 5 December 2011
Received in revised form 15 February 2012
Available online 19 May 2012

Keywords:
Stock price model
Continuum percolation
Statistical analysis
Tail distribution
Lyapunov exponent
Zipf analysis
Computer simulation

A B S T R A C T

The continuum percolation system is developed to model a random stock price process in this work. Recent empirical research has demonstrated various statistical features of stock price changes, the financial model aiming at understanding price fluctuations needs to define a mechanism for the formation of the price, in an attempt to reproduce and explain this set of empirical facts. The continuum percolation model is usually referred to as a random coverage process or a Boolean model, the local interaction or influence among traders is constructed by the continuum percolation, and a cluster of continuum percolation is applied to define the cluster of traders sharing the same opinion about the market. We investigate and analyze the statistical behaviors of normalized returns of the price model by some analysis methods, including power-law tail distribution analysis, chaotic behavior analysis and Zipf analysis. Moreover, we consider the daily returns of Shanghai Stock Exchange Composite Index from January 1997 to July 2011, and the comparisons of return behaviors between the actual data and the simulation data are exhibited.

1. Introduction

The statistical analysis of financial market index and return is an active topic to understand and model the distribution of financial price fluctuation, which has long been a focus of economic research. As the stock markets are becoming deregulated worldwide, the modeling of dynamics of forwards prices is becoming a key problem in the risk management, physical assets valuation, and derivatives pricing, and it is also important to understand the statistical properties of fluctuations of stock prices in globalized securities markets. With the emerging of new statistical analyzing methods and computer-intensive analyzing methods in the last decade, the empirical results of stock returns provide various empirical evidence that has challenged the old random-walk hypothesis, requiring the invention of new financial models to describe price movements in the market. A series of statistical behaviors, the so-called "stylized facts", such as fat tails phenomenon of price changes, power-law distributions of logarithmic returns and volume, volatility clustering of absolute returns and multifractality of volatility, are revealed from empirical research by previous studies [1–5]. In an attempt to reproduce and explain these stylized facts, various market models have been introduced, some of which approach in this field by considering interacting particle systems [6–13]. Stauffer and Penna [8] and Tanaka [9] developed a price model by the lattice percolation system [14–17], the local interaction or influence among traders in a stock market is constructed, and a cluster of percolation is applied to define the cluster of traders sharing the same opinion about the market. They suppose that the spread of information leads to the stock price fluctuation, and when the influence rate of the model is around or at a critical value, the existence of fat-tail behavior for the returns is clearly observed. The critical phenomena of percolation model is used to illustrate the herd behavior of stock market participants. Zhang and Wang [12] invented the finite-range contact particle

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0378-4371/$ – see front matter
doi:10.1016/j.physa.2012.05.024

system to model a stock price process for studying the behaviors of returns by statistical analysis and computer simulation. The epidemic spreading of the contact model is considered as the spreading of the investors' investment attitudes towards the stock market, and supposes that the investment attitudes are represented by the viruses of the contact model, which accordingly classify buying stock, selling stock and holding stock.

In the present paper, we present a financial price model by applying the continuum percolation system. The continuum percolation model is usually referred to as a random coverage process or a Boolean model, it is a member of a class of stochastic processes known as interacting particle systems. In this financial model, the local interaction or influence among traders is developed by the continuum percolation, and a cluster of continuum percolation is applied to define the cluster of traders sharing the same opinion about the market. Then we study the power-law tails behavior, chaotic behavior and Zipf distribution of returns of the actual data and the simulation data by statistical analysis and computer simulation. We select the actual data for the closing prices of each trading day of Shanghai Stock Exchange Composite Index (SSE) in a fourteen and half-year period from 1 January 1997 to 29 July 2011, the total number of observed data for SSE is about 3525, and the database of SSE is from the website www.sse.com.cn.

2. Financial price model with continuum percolation system

2.1. Description of continuum percolation system

In this section, we give a brief description of the continuum percolation system, for more details see Refs. [15,18–20]. One interpretation of the continuum percolation, is often thought of as a crude model for the process of a ground getting wet during a period of rain. The area getting wet depends on the place where raindrops fall on the ground and the size of the wetted region per raindrop. We now introduce the mathematical construction of the continuum percolation model, which is usually referred to as Boolean model in stochastic geometry. Consider a d-dimensional Euclidean space $\mathbb{R}^d$, denote the σ-algebra of Borel sets in $\mathbb{R}^d$ by $\mathscr{B}^d$, and denote by N the set of all counting measures on $\mathscr{B}^d$ which assign finite measure to bounded Borel sets with the measure of a point no larger than 1. Then, we equip N with the σ-algebra $\mathscr{N}$ generated by sets of the form $\mathscr{N} = \{n \in N : n(A) = k\}$, where $A \in \mathscr{B}^2$ and k is an integer. A point process X is a measurable mapping from a probability space $(\Omega, \mathcal{F}, P)$ into $(N, \mathscr{N})$. $X(A)$ represents the random number of points inside A. The point process X is said to be a homogeneous Poisson process with density $\lambda > 0$ if it satisfies the following: (i) For mutually disjoint Borel sets $A_1, \ldots, A_k$, the random variables $X(A_1), \ldots, X(A_k)$ are mutually independent; (ii) For any bounded Borel set $A \in \mathscr{B}^d$ we have for every $k \geq 0$, $P(X(A) = k) = e^{-\lambda\ell(A)}\lambda^k\ell(A)^k/k!$, where $\ell(\cdot)$ denotes the Lebesgue measure in $\mathbb{R}^d$.

The points of a homogeneous Poisson process with intensity λ are described by a countable collection of random vectors $X_1, X_2, \ldots$. We put a sphere of radius ρ_i with center at each point X_i, denote by P_i for each $i \geq 1$. And we are interested in the shapes of clusters of overlapping spheres, and particularly in the possible existence of infinite clusters. It is said that percolation occurs if, with positive probability, any given random sphere is part of an infinite clump of random spheres. Two spheres P_i and P_j are called adjacent if $P_i \cap P_j \neq \varnothing$. We write $P_i \leftrightarrow P_j$ if there exists a sequence $P_{i_1}, P_{i_2}, \ldots, P_{i_k}$ of spheres such that $P_{i_1} = P_i$ and $P_{i_k} = P_j$, and P_{i_q} is adjacent to $P_{i_{q+1}}$ for $1 < q < k$, see Fig. 1. The size of a cluster is thus the number of spheres belonging to it. $C(X_i)$ denotes a cluster containing sphere P_i, and $|C(X_i)|$ represents its size. If there exists a $C(X_i)$ such that $|C(X_i)| = \infty$ (there is a infinite cluster), we say that percolation occurs. From the continuum percolation theory, we know that there is a non-trivial critical value of λ_c, such that the infinite cluster always exists on $\mathbb{R}^2$ when $\lambda > \lambda_c$. Let $\theta_\rho(\lambda) = \theta(\lambda)$ be the probability that the origin is an element of an unbounded occupied component, then the function $\theta_\rho(\lambda)$ is called the percolation function. We define the critical value $\lambda_c = \lambda_c(\rho)$ as follows:

$$\lambda_c(\rho) = \inf\{\lambda : \theta_\rho(\lambda) > 0\}.$$

2.2. Financial price model

In this section, the continuum percolation system is employed to model the financial price process. Consider a model of auctions for a stock in a financial market, we derive the stock price process from the auctions. We assume that the traders are arranged at the Poisson points $X_1, X_2, \ldots$, the centers of spheres in a finite lattice rectangle $[-l, l]^2$. And suppose that each investor can take three trading positions: buying position, selling position or neutral position. Generally, the investors decide their trading positions by analyzing past market data, investment environment and their trading strategy. And the investing attitudes towards to the financial market are exchanged among the market participants. As is demonstrated by Cont and Bouchaud [1], the interaction and communication among investors is an essential ingredient of market organization. In real markets, groups of traders may share information and align their decisions and act in unison to buy or sell, this phenomenon is thought to be the "herd effect" of investors in the market. Therefore, it is important to consider not only the strategies of individual investors but also the interaction between investors. We further assume that when the distance between any two investors is less than a certain value d (which we call the sight of the investor), these two investors are called neighbors and they are able to exchange information about the market with each other. It is obvious that the sight of investor is equivalent to the diameter of the sphere in continuum percolation theory, that is, $d = 2\rho$. In this way, information can pass on from neighbor to neighbor and consequently investors group into clusters through neighboring relations, as described in continuum percolation theory in Section 2.1 (see Fig. 1). Further, we suppose that once a cluster of investors has been