Int. J. Production Economics 131 (2011) 528–534

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Closed-form solutions for Wee's and Martin's EOQ models with a temporary price discount

Horng-Jinh Chang [a], Wen-Feng Lin [b,*], Jow-Fei Ho [a]

[a] Graduate Institute of Management Sciences, Tamkang University, Taipei, Taiwan
[b] China University of Science and Technology, Taipei, Taiwan

ARTICLE INFO

Article history:
Received 4 January 2008
Accepted 24 December 2010
Available online 4 February 2011

Keywords:
Deterioration
Economic order quantity
Temporary sale price
Discount price

ABSTRACT

In this article, we use closed-form solutions to solve Wee and Yu (1997) deteriorating inventory model with a temporary price discount and Martin's (1994) EOQ model with a temporary sale price. In Wee and Yu (1997) and Martin's (1994), the benefits during the temporary price discount purchase cycle are represented by their objective functions. Wee and Yu (1997) and Martin (1994) only used search methods to find approximate solutions. Following the theorems we suggested, you can find closed-form solution directly when there are integer operators involve in an objective function. Using the data of Wee and Yu (1997) and Martin (1994), we can find the results are more quick and more accurate.

1. Introduction

In the real life, the manufacturer increases the sales volume and profit by temporarily reducing price. Ghare and Schrader (1963) first considered inventory model for items deteriorating at a constant rate. They derived an EOQ model for an exponentially decaying inventory. Tersine (1994) proposed a temporary price discount model, the optimal EOQ policy is obtained by maximizing the difference between regular EOQ cost and special quantity cost during the sale period. But in Tersine's (1994) article, the average inventory was represented as $0.5Q^*$ deserved something to discuss. Martin (1994) did average inventory appropriate correction. But Tersine (1994) and Martin (1994) did not consider the fact that some commodities may deteriorate with time. Wee and Yu (1997) considered the items deteriorated exponentially with time when temporary price discount purchase occurs at the regular and non-regular replenishment time. Martin (1994) and Wee and Yu (1997) sacrificed the closed-form solutions in solving their objective functions, instead of using search methods to find special order quantity and maximum gain. Wee et al. (2003) showed that Tersine's (1994) EOQ optimal solution could be derived algebraically without using differential calculus. Saker and Kindi (2006) proposed five different cases of the discount sale scenarios in order to maximize the annual gain of the special ordering quantity. Cárdenas-Barrón (2009a) extended Saker and Kindi (2006) to determine the optimal ordering policies when a supplier establishes a minimum order size. Cárdenas-Barrón (2009b) pointed out that there are some technical and mathematical expression errors in Saker and Kindi (2006) and presented the closed form solutions for the optimal total gain cost. Li (2009) presented a new method for determining the optimal number of orders for the finite-horizon discrete-time EOQ model. García-Laguna et al. (2010) presented methods to obtain solutions of the EOQ and EPQ models when the lot sizes are integer variables to be determined. The related analysis on global optimization has been performed by Abad (2003), Chung and Wee (2008), Chung et al. (2008), Wee et al. (2009), Yang et al. (2010), Cárdenas-Barrón et al. (2010), *etc.*

The remainder of this paper is organized as follows. In Section 2, we describe the notation which is used throughout this paper. In Section 3, we describe the mathematical models and suggest theorems to determine the optimal ordering policy. Numerical examples are provided in Section 4. Finally, we make a conclusion in the last section.

2. Notation

$\lceil\ \rceil$	integer operator, integer value equal to or greater than its argument
$\lfloor\ \rfloor$	integer operator, integer value equal to or less than its argument
P	unit purchase cost before the discount.
d	unit price discount
C	cost per order
F	annual holding cost fraction
R	annual demand

* Corresponding author. Tel.: +886 2 27867048; fax: +886 2 27867253.
E-mail address: linwen@cc.cust.edu.tw (W.-F. Lin).

0925-5273/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2011.01.021

θ	deterioration rate
t_1	time between temporary price discount replenishment epoch and the next. regular replenishment epoch
t_2	time between the last regular replenishment epoch and the end of $\hat{T}$
T^*	optimal replenishment cycle for a regular price order, $T^* = \sqrt{2C/PR(\theta+F)}$
T^0	optimal replenishment cycle for a discount price with no special order, $T^0 = \sqrt{P/(P-d)}T^*$
$\hat{T}$	optimal temporary price discount replenishment cycle
$\hat{Q}_0$	the special order quantity during the temporary price discount purchase cycle
Q_0^*	the regular EOQ lot size, $Q_0^* = \sqrt{2CR/FP}$
TC_{si}	the total cost when the special order is taken, $i=1,2,3$
TC_{ni}	the total cost during the same period when the standing EOQ policy is retained, $i=1,2,3$
$G_i(\hat{T})$	the benefit during the temporary price discount purchase cycle for Wee's model, $i=1,2$
$G_3(\hat{Q}_0)$	the benefit during the temporary price discount purchase cycle for Martin's model
$\hat{T}_{m1}(m)$	the local maximum point of $G_1(\hat{T})$ between $\hat{T}=T^0+mT^*$ and $\hat{T}=T^0+(m+1)T^*$
$\hat{T}_{m2}(m)$	the local maximum point of $G_2(\hat{T})$ between $\hat{T}=t_1+mT^*$ and $\hat{T}=t_1+(m+1)T^*$
$\hat{Q}_{m3}(m)$	the local maximum point of $G_3(\hat{Q}_0)$ between $\hat{Q}_0=mQ_0^*$ and $\hat{Q}_0=(m+1)Q_0^*$
$GM_1(m)$	the maximum value of $G_1(\hat{T})$ between $\hat{T}=T^0+mT^*$ and $\hat{T}=T^0+(m+1)T^*$
$GM_2(m)$	the maximum value of $G_2(\hat{T})$ between $\hat{T}=t_1+mT^*$ and $\hat{T}=t_1+(m+1)T^*$
$GM_3(m)$	the maximum value of $G_3(\hat{Q}_0)$ between $\hat{Q}_0=mQ_0^*$ and $\hat{Q}_0=(m+1)Q_0^*$

3. Mathematical models

3.1. Wee's case 1 model: temporary price discount price order occurs at regular replenishment time

The special order and standing EOQ policy of Wee and Yu (1997) *case* 1 model are shown in Fig. 1.

Eq. (8) in Wee and Yu (1997) represented the total cost when the special order is taken.

$$TC_{s1} \approx C+R(P-d)\left(\hat{T}+\frac{\theta\hat{T}^2}{2}\right)+\frac{(P-d)FR\hat{T}^2}{2} \quad (1)$$

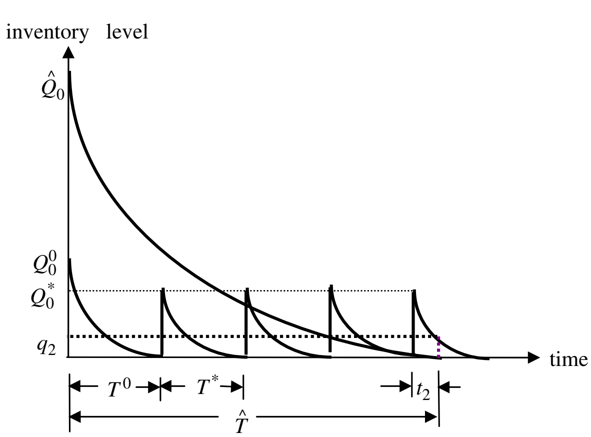

Fig. 1. Wee's *case* 1 model.

And Eq. (19) in Wee and Yu (1997) represented the total cost during the same time without special price order.

$$\begin{aligned} TC_{n1} \approx & \left(\left\lceil\frac{\hat{T}-T^0}{T^*}\right\rceil+1\right)C+(P-d)R\left(T^0+\frac{\theta T^{02}}{2}\right) \\ & +PR\left(T^*+\frac{\theta T^{*2}}{2}\right)\left\lceil\frac{\hat{T}-T^0}{T^*}\right\rceil+\frac{1}{2}(P-d)FRT^{02}+\frac{1}{2}PFRT^{*2}\left\lfloor\frac{\hat{T}-T^0}{T^*}\right\rfloor \\ & +PFR\left[T^*\left(\hat{T}-T^0-T^*\left\lfloor\frac{\hat{T}-T^0}{T^*}\right\rfloor\right)-\frac{1}{2}\left(\hat{T}-T^0-T^*\left\lfloor\frac{\hat{T}-T^0}{T^*}\right\rfloor\right)^2\right] \\ & -PR\left\{\left[T^*-\left(\hat{T}-T^0-T^*\left\lfloor\frac{\hat{T}-T^0}{T^*}\right\rfloor\right)\right]\right. \\ & \left.+\frac{1}{2}\theta\left[T^*-\left(\hat{T}-T^0-T^*\left\lfloor\frac{\hat{T}-T^0}{T^*}\right\rfloor\right)\right]^2\right\} \end{aligned} \quad (2)$$

The benefit during the temporary price discount purchase cycle is

$$G_1(\hat{T}) = TC_{n1}-TC_{s1} \quad (3)$$

$$\begin{aligned} G_1(\hat{T}) = & \left\lceil\frac{\hat{T}-T^0}{T^*}\right\rceil C+(P-d)R\left(T^0+\frac{\theta T^{02}}{2}\right)+PR\left(T^*+\frac{\theta T^{*2}}{2}\right)\left\lceil\frac{\hat{T}-T^0}{T^*}\right\rceil \\ & +\frac{1}{2}(P-d)FRT^{02}+\frac{1}{2}PFRT^{*2}\left\lfloor\frac{\hat{T}-T^0}{T^*}\right\rfloor \\ & +PFR\left[T^*\left(\hat{T}-T^0-T^*\left\lfloor\frac{\hat{T}-T^0}{T^*}\right\rfloor\right)-\frac{1}{2}\left(\hat{T}-T^0-T^*\left\lfloor\frac{\hat{T}-T^0}{T^*}\right\rfloor\right)^2\right] \\ & -PR\left\{\left[T^*-\left(\hat{T}-T^0-T^*\left\lfloor\frac{\hat{T}-T^0}{T^*}\right\rfloor\right)\right]\right. \\ & \left.+\frac{1}{2}\theta\left[T^*-\left(\hat{T}-T^0-T^*\left\lfloor\frac{\hat{T}-T^0}{T^*}\right\rfloor\right)\right]^2\right\} \\ & -R(P-d)\left(\hat{T}+\frac{\theta\hat{T}^2}{2}\right)-\frac{FR(P-d)\hat{T}^2}{2} \end{aligned} \quad (4)$$

The temporary price discount replenishment is justified only in the following condition:

$$\hat{Q}_0 > \frac{R}{\theta}\left\{\left[Q_0^*\left(\frac{\theta}{R}\right)+1\right]^{\sqrt{P/P-d}}-1\right\} \quad (5)$$

Property 1. $G_1(\hat{T})$ is a piecewise continuous function. The jump value of $G_1(\hat{T})$ at $\hat{T}=T^0+mT^*$ is C, where m is a positive integer.

Property 2. m is a positive integer.

Let

$$m_{1L} = \frac{d\sqrt{PR}-(P-d)\sqrt{2C(\theta+F)}-\sqrt{2PC(\theta+F)(P-d)}}{(P-d)\sqrt{2C(\theta+F)}} \quad (6)$$

$$m_{1R} = \frac{d\sqrt{PR}+P\sqrt{2C(\theta+F)}-\sqrt{2PC(\theta+F)(P-d)}}{(P-d)\sqrt{2C(\theta+F)}} \quad (7)$$

(i) (i) $G_1(\hat{T})$ is an increasing function of $\hat{T}$ between T^0+mT^* and $T^0+(m+1)T^*$ when $m<\lceil m_{1L}\rceil$.
(ii) (ii) $G_1(\hat{T})$ is a decreasing function of $\hat{T}$ between T^0+mT^* and $T^0+(m+1)T^*$ when $m>\lceil m_{1R}\rceil$.
(iii) (iii)$G_1(\hat{T})$ is a concave function of $\hat{T}$ between T^0+mT^* and $T^0+(m+1)T^*$ when $\lceil m_{1L}\rceil \le m \le \lfloor m_{1R}\rfloor$.

Proofs of Properties 1 and 2 are given in appendix. Now we propose Theorem 1 to find the optimal replenishment cycle $\hat{T}$ and the global maximum value of $G_1(\hat{T})$ for Wee's *case* 1 model.