Digital Signal Processing 23 (2013) 817–830

Contents lists available at SciVerse ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

A unified approach to tracking performance analysis of the selective partial update adaptive filter algorithms in nonstationary environment

Mohammad Shams Esfand Abadi *, Fatemeh Moradiani

Faculty of Electrical and Computer Engineering, Shahid Rajaee Teacher Training University, P.O. Box 16785-163, Tehran, Iran

A R T I C L E I N F O

Article history:
Available online 27 December 2012

Keywords:
Adaptive filter
Selective partial update
Nonstationary
Mean-square performance
Energy conservation

A B S T R A C T

In this paper, a unified approach to mean-square performance analysis of the family of selective partial update (SPU) adaptive filter algorithms in nonstationary environment is presented. Using this analysis, the tracking performance of Max normalized least mean squares (Max-NLMS), *N*-Max NLMS, the various types of SPU-NLMS algorithms, SPU transform domain LMS (SPU-TD-LMS), the family of SPU affine projection algorithms (SPU-APA), the family of selective regressor APA (SR-APA), the dynamic selection of APA (DS-APA), the family of SPU-SR-APA, the family of SPU-DS-APA, SPU subband adaptive filters (SPU-SAF), and the periodic, sequential, and stochastic partial update LMS, NLMS, and APA as well as classical adaptive filter algorithms can be analyzed with a unified approach. Two theoretical expressions are introduced to study the performance. The analysis is based on energy conservation arguments and does not need to assume a Gaussian or white distribution for the regressors. We demonstrate through simulations that the derived expressions are useful in predicting the performance of this family of adaptive filters in nonstationary environment.

1. Introduction

Adaptive filtering is an important subfield of digital signal processing which has been actively researched for more than four decades [1–3]. The adaptive filters are able to track the variations of the input signal properties. The purpose of tracking performance analysis is to characterize the tracking ability of adaptive filters in nonstationary environments. In this area of research, the transient performance of least mean squares (LMS) was presented in [4], and [5] for two models of optimal weight vector. Also, the steady-state performance of LMS algorithm in nonstationary environment was analyzed in [6]. Different researches for the signed regressor LMS algorithm can be found in [7–10]. A more general analysis for various adaptive filter algorithms was introduced in [11]. In [12], a unified approach for tracking performance of the family of affine projection algorithm (APA) for an independent and identically distributed input signal was presented. The approach of [12] was based on energy-conservation relation which was originally derived in [13] and [14]. This approach was successfully extended to transform domain and subband adaptive filters [15,16].

There is another class of adaptive filters, where the filter coefficients are partially updated. These algorithms update only a subset of filter coefficients in each iteration. In contrast to full update adaptive algorithms, the performance analysis of adaptive filters with selective partial updates (SPU) has not been widely studied in nonstationary environments. Many theoretical performance analyses have been presented in the literature independently. Moreover, most of the analyses were introduced for the specific input signal in a stationary environment [17–19]. The general performance analysis for the family of SPU normalized LMS (SPU-NLMS) algorithms in the stationary environment can be found in [20]. In [21], the tracking performance of some SPU adaptive filter algorithms was studied. But the analysis was presented for the white Gaussian input signal. Recently, the mean-square performance of SPU-NLMS and SPU-APA was analyzed in [22] for nonstationary environment. The approach of [22] was based on energy relation.

What we propose here is a unified approach to tracking performance analysis of the family of SPU-NLMS algorithms. Based on this approach, the performance of Max-NLMS [23], *N*-Max NLMS [24,17] (*N* is the number of filter coefficients to update), the variants of the SPU-NLMS [25,18,19,26], SPU-TD-LMS [27], SPU-SAF [28], the family of SPU-APA [18], SR-APA [29], DS-APA [30], SPU-SR-APA [31], SPU-DS-APA, the periodic, sequential, and stochastic partial update LMS, NLMS, and AP algorithms [32,26] can be studied in nonstationary environment. Also, the tracking performance of classical adaptive filters can be studied according to the presented analysis. The analysis is based on energy conservation arguments and does not need to assume the Gaussian or white distribution for the regressors [3].

This paper is organized as follows. In Section 2, a generic update equation for the family of SPU adaptive filter algorithms is

* Corresponding author. Fax: +98 21 22970003.
E-mail addresses: mshams@srttu.edu (M.S.E. Abadi), f20m-am@yahoo.com (F. Moradiani).

1051-2004/$ – see front matter
http://dx.doi.org/10.1016/j.dsp.2012.12.012

Table 1
The most common families of adaptive filter algorithms can be described through $\mathbf{h}(n+1)=\mathbf{h}(n)+\mu\mathbf{C}(n)\mathbf{X}(n)\mathbf{W}(n)\mathbf{e}(n)$.

Algorithm	P	D	$\mathbf{C}(n)$	$\mathbf{W}(n)$
LMS	$P=1$	$D=1$	$\mathbf{I}$	$\mathbf{I}$
NLMS	$P=1$	$D=1$	$\mathbf{I}$	$1/\Vert\mathbf{x}(n)\Vert^2$
ϵ-NLMS	$P=1$	$D=1$	$\mathbf{I}$	$(\epsilon+\mathbf{x}^T(n)\mathbf{x}(n))^{-1}$
APA	$P\leqslant M$	$D=1$	$\mathbf{I}$	$(\mathbf{X}^T(n)\mathbf{X}(n))^{-1}$
BNDR-LMS	$P=2$	$D=1$	$\mathbf{I}$	$(\mathbf{X}^T(n)\mathbf{X}(n))^{-1}$
R-APA	$P\leqslant M$	$D=1$	$\mathbf{I}$	$(\epsilon\mathbf{I}+\mathbf{X}^T(n)\mathbf{X}(n))^{-1}$
NLMS-OCF	$P\leqslant M$	$D\geqslant 1$	$\mathbf{I}$	$(\mathbf{X}^T(n)\mathbf{X}(n))^{-1}$
DR-LMS	$P\leqslant M$	$D=1$	$\mathbf{I}$	$\mathbf{I}$
NDR-LMS	$P\leqslant M$	$D=1$	$\mathbf{I}$	$\mathrm{diag}\{1/\Vert\mathbf{x}(n)\Vert^2,\dots,1/\Vert\mathbf{x}(n-P+1)\Vert^2\}$
RLS	$P=1$	$D=1$	$[\sum_{i=0}^{n}\lambda^{n-i}\mathbf{x}(i)\mathbf{x}^T(i)]^{-1}$	$\mathbf{I}$
TDAF	$P=1$	$D=1$	$\mathbf{T}.[\mathrm{diag}\{\mathrm{diag}\{\sum_{i=0}^{n}\lambda^{n-i}\mathbf{T}^T\mathbf{x}(i)\mathbf{x}^T(i)\mathbf{T}\}\}]^{-1}.\mathbf{T}^T$	$\mathbf{I}$

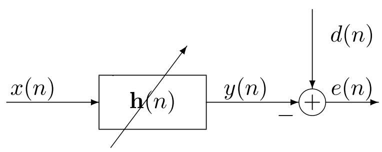

Fig. 1. Prototypical adaptive filter setup.

introduced. Sections 3 and 4 describe the derivation of the classical and SPU adaptive filters. In the next section, the mean-square performance analysis is presented in nonstationary environment and two theoretical expressions are introduced. We conclude the paper by showing a comprehensive set of simulations supporting the validity of the results.

Throughout the paper, the following notations are used:

$\Vert.\Vert^2$	Squared Euclidean norm of a vector.
$\Vert\mathbf{t}\Vert^2_{\mathbf{\Sigma}}$	$\mathbf{\Sigma}$-weighted Euclidean norm of a column vector $\mathbf{t}$ defined as $\mathbf{t}^T\mathbf{\Sigma}\mathbf{t}$.
$\mathrm{vec}(\mathbf{T})$	An $M^2\times 1$ column vector $\mathbf{t}$ obtained by stacking the columns of the $M\times M$ matrix $\mathbf{T}$.
$\mathrm{vec}^{-1}(\mathbf{t})$	Creating an $M\times M$ matrix $\mathbf{T}$ from the $M^2\times 1$ column vector $\mathbf{t}$.
$\mathbf{A}\otimes\mathbf{B}$	Kronecker product of matrices $\mathbf{A}$ and $\mathbf{B}$.
$E\{.\}$	Expectation operator.
$(.)^T$	Transpose of a vector or a matrix.
diag(.)	Has the same meaning as the MATLAB operator with the same name: If its argument is a vector, a diagonal matrix with the diagonal elements given by the vector argument results. If the argument is a matrix, its diagonal is extracted into a resulting vector.

2. The generic adaptive filter update equation

Fig. 1 shows a typical adaptive filter setup, where $x(n)$, $d(n)$ and $e(n)$ are the input, the desired and the output error signals, respectively. Here, $\mathbf{h}(n)$ is the $M\times 1$ column vector of filter coefficients at iteration n.

The generic filter vector update equation at the center of our analysis is introduced as

$$\mathbf{h}(n+1)=\mathbf{h}(n)+\mu\mathbf{C}(n)\mathbf{X}(n)\mathbf{W}(n)\mathbf{e}(n), \qquad (1)$$

where

$$\mathbf{e}(n)=\mathbf{d}(n)-\mathbf{X}^T(n)\mathbf{h}(n) \qquad (2)$$

is the output error vector. The matrix $\mathbf{X}(n)$ is the $M\times P$ input signal matrix

$$\mathbf{X}(n)=\big[\mathbf{x}(n),\mathbf{x}(n-D),\dots,\mathbf{x}\big(n-(P-1)D\big)\big], \qquad (3)$$

where P is a positive integer (usually, but not necessarily $P\leqslant M$), D is the positive integer parameter ($D\geqslant 1$) that can increase the separation, and consequently reduce the correlation among the regressors in $\mathbf{X}(n)$, $\mathbf{x}(n)=[x(n),x(n-1),\dots,x(n-M+1)]^T$ is the input signal vector, and $\mathbf{d}(n)$ is a $P\times 1$ vector of desired signal

$$\mathbf{d}(n)=\big[d(n),d(n-D),\dots,d\big(n-(P-1)D\big)\big]^T. \qquad (4)$$

The desired signal is assumed to be generated from the following linear model

$$\mathbf{d}(n)=\mathbf{X}^T(n)\mathbf{h}_t(n)+\mathbf{v}(n), \qquad (5)$$

where $\mathbf{v}(n)=[v(n),v(n-D),\dots,v(n-(P-1)D)]^T$ is the measurement noise vector. The random noise $v(n)$ is assumed zero mean, independent and identically distributed, and statistically independent of the input signal matrix, and $\mathbf{h}_t(n)$ is the unknown filter vector which is time-variant. We assume that the variation of $\mathbf{h}_t(n)$ is according to the random walk model [3,12]

$$\mathbf{h}_t(n+1)=\mathbf{h}_t(n)+\mathbf{q}(n), \qquad (6)$$

where the sequence of $\mathbf{q}(n)$ is a zero mean, an independent and identically distributed sequence with autocorrelation matrix $\mathbf{Q}=E\{\mathbf{q}(n)\mathbf{q}^T(n)\}$ and independent of the $\mathbf{x}(k)$ for all k and of the $d(k)$ for $k<n$. Also, the variance of $\mathbf{q}(n)$ is assumed small in order to obtain slow variation in $\mathbf{h}_t(n)$.

3. Derivation of classical adaptive filter algorithms

We can now make specific choices for the matrices $\mathbf{C}(n)$ and $\mathbf{W}(n)$ as well as for the parameters P and D. Different adaptive filter algorithms can be viewed as specific instantiations of the generic adaptive filter update equation [33].

3.1. Derivation of LMS, NLMS, and ϵ-NLMS algorithms

By setting the parameters P and D equal to 1, and substituting the matrices $\mathbf{C}(n)$ and $\mathbf{W}(n)$ from Table 1 in the generic adaptive filter update equation, LMS, NLMS, and ϵ-NLMS[1] are established respectively.

3.2. Derivation of the family of affine projection and data-reusing algorithms

From the generic adaptive filter update equation and the parameters selection according to Table 1, the family of affine projection algorithms will be established. These algorithms are the standard version of APA, the regularized APA (R-APA) [34], the binormalized data-reusing LMS (BNDR-LMS) [35], the data-reusing adaptive algorithms such as the data-reusing LMS (DR-LMS), the normalized DR-LMS (NDR-LMS) [36], and the NLMS with orthogonal correction factors (NLMS-OCF) [37].

[1] The parameter ϵ is the regularization parameter.