Digital Signal Processing 23 (2013) 797–807

Contents lists available at SciVerse ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

Performance analysis on Lv distribution and its applications

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ARTICLE INFO

Article history:
Available online 20 December 2012

Keywords:
Lv distribution
Signal distribution concentration
Signal detection
Local polynomial periodogram

ABSTRACT

A new signal analysis method, known as Lv distribution (LVD), has been reported recently to provide improved estimation accuracy of centroid frequency and chirp rate. In this paper, performances of the LVD on signal concentration, detection, representation errors and computational complexity are discussed and compared with polynomial Fourier transform (PFT) and fractional Fourier transform (FrFT). Based on the results of our theoretical analysis and Monte Carlo simulations, it is shown that the LVD achieves desirable performance improvement compared with those achieved by other methods. By using the accurate estimation of chirp rate provided by the LVD, the performance of local polynomial periodogram (LPP) is investigated. Comparisons with other time–frequency representations, such as the inverse LVD (ILVD) and the PFT-based LPP, are made on signal concentration in the time–frequency domain.

1. Introduction

Time–frequency analysis has been widely recognized to be a useful tool to reveal the characteristics of signal frequencies varying with time for many practical applications, such as geology, music, wireless communication and radar [1–3]. Many reported methods detecting the widely used linear frequency modulated (LFM) signals in literature are mainly classified into the categories of time–frequency representation (TFR) and frequency–chirp rate representation (FCR). The TFR, such as short-time Fourier transform (STFT) [4], produces spectrograms to display the frequency variation with time. As a generalized form of the STFT, the local polynomial periodogram (LPP) is able to more accurately describe the time–frequency characteristics by using estimated parameters [5,6]. To improve signal distribution concentration, various transforms in the Cohen class have been proposed. The most well known Wigner–Ville distribution (WVD) is able to achieve the best signal concentration if the input is a single-component LFM signal. As the WVD generally suffers from cross-terms for multi-component signals, many methods, such as smoothed pseudo WVD [7], S-method [8] and scaled L-Wigner distribution [9], have been reported to minimize the effects of cross-terms. Although some desirable improvements on this issue have been made, the trade-off between auto-terms and cross-terms always exists, which often limits further improvement on signal concentration in the time–frequency domain.

Another category of the methods dealing with LFM signals is the FCR. For example, the polynomial Fourier transform (PFT) [10,11] and fractional Fourier transform (FrFT) [12] are able to provide information of signal frequencies and chirp rates. Among these methods, the FrFT has been receiving increasing attention because of its linearity, i.e., no cross-term, and relatively better signal concentration. Since the LFM signal is distributed into a line on the TFR, some integral-based methods, including the Radon–Wigner transform [13,14], Radon-ambiguity transform [15], Wigner–Hough transform [16] and LPP-Hough transform [17] have been reported to obtain parameter estimation. In general, better processing performance has been achieved by using much more computational complexity for the Hough transform.

The recently reported Lv distribution (LVD) [18] belongs to the category of FCR. Different from the FrFT that uses rotation angles to indirectly obtain the chirp rate, the LVD directly provides accurate representations of LFM signals with centroid frequencies and chirp rates by applying auto-correlation, scaling operation and Fourier transform (FT). Furthermore, the LVD is asymptotic linear so that a signal consisting of multiple LFM components can be represented without much undesirable cross-terms. It is also shown that the LVD is able to achieve a good estimation performance in low signal-to-noise ratio (SNR) environments.

Although the performances of LVD have been partially analyzed in [18], its desirable capability of representing LFM signals has not been fully investigated and its merits have not been compared with these of other commonly used methods. This paper presents performance analysis in terms of some important issues, such as signal concentration and detection, representation errors and computational complexity of the LVD, which are essential for practical applications. For example, the comparisons with FrFT and PFT are

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1051-2004/$ – see front matter
http://dx.doi.org/10.1016/j.dsp.2012.11.011

particularly relevant to the selection of a suitable signal representation for a particular application. The results presented in this paper show that the LVD can achieve better performance compared to those by the FrFT and PFT. Since the LVD is asymptotic linear, similar to the STFT/FrFT, and deals with auto-correlation of signals, similar to the WVD, it can be considered as a desirable combination of linear and bilinear methods to make use of both advantages of trivial cross-terms and high signal concentration. Finally, the LPP, as a TFR, by using the chirp rate estimated by the LVD is also re-examined in this paper.

The main contributions of this paper are as follows:

(i) Analysis of signal distribution concentration in the LVD domain theoretically and experimentally to provide performance benchmarks for noisy multi-component LFM signals.
(ii) Analysis of LFM signal detection performance of the LVD based on hypothesis testing, providing detection method and corresponding threshold.
(iii) Comparison and discussion of the performances achieved by the LVD, PFT and FrFT.
(iv) Performance improvement made by the LPP based on the chirp rates estimated by the LVD.

The rest of this paper is organized as follows. Section 2 briefly reviews the LVD for convenience of readers. Important performance analysis, in terms of signal distribution, signal detection, representation errors and computational complexity, is discussed in Section 3. In Section 4, an application of the LVD on chirp rate estimation for the LPP is reported. Some conclusions are presented in Section 5.

2. Review on the LVD [18]

For the convenience of readers, this section briefly reviews the definition of LVD and its computation details. More information on the LVD is available in [18]. Let us consider a multi-component LFM signal expressed as

$$\begin{aligned} x(t) = s(t) + v(t) &= \sum_{k=1}^{K} s_k(t) + v(t) \\ &= \sum_{k=1}^{K} A_k \exp\bigl(j2\pi f_k t + j\pi\gamma_k t^2\bigr) + v(t), \end{aligned} \tag{1}$$

where K is the number of LFM components, A_k, f_k and γ_k, $k = 1, \dots, K$, are their constant amplitudes, centroid frequencies and chirp rates, respectively, and $v(t) \sim N(0, \sigma_v^2)$ is additive white Gaussian noise. Its parametric symmetric instantaneous auto-correlation function (PSIAF) is defined as

$$\begin{aligned} R_x^C(t,\tau) &= x\left(t + \frac{\tau + a}{2}\right) x^*\left(t - \frac{\tau + a}{2}\right) \\ &= \sum_{k=1}^{K} A_k^2 \exp\bigl(j2\pi f_k(\tau + a) + j2\pi\gamma_k(\tau + a)t\bigr) \\ &\quad + \sum_{i=1}^{K-1}\sum_{j=i+1}^{K} \bigl(R_{s_i s_j}^C(t,\tau) + R_{s_j s_i}^C(t,\tau)\bigr) \\ &\quad + \sum_{k=1}^{K} \bigl(R_{s_k v}^C(t,\tau) + R_{v s_k}^C(t,\tau)\bigr) + R_v^C(t,\tau), \end{aligned} \tag{2}$$

where a denotes a constant time-delay, $R_{s_i s_j}^C$ represents the cross-terms between different LFM components, $R_{s_k v}^C$ and $R_{v s_k}^C$ denote the cross-terms between the signal components and noise, and R_v^C represents the noise auto-term. We define a scaling operator Γ of a phase function G with respect to (t, τ) as

$$\Gamma\bigl[G(t,\tau)\bigr] \to G\left(\frac{t_n}{h(\tau + a)}, \tau\right), \tag{3}$$

where h is a scaling factor and t_n is the scaled time expressed as $t_n = (\tau + a)ht$. According to [18], we generally use the parameters $a = 1$ and $h = 1$ for obtaining a desirable FCR. Let us perform the scaling operation Γ on the PSIAF in (2) to obtain

$$\begin{aligned} \Gamma\bigl[R_x^C(t,\tau)\bigr] &= \sum_{k=1}^{K} A_k^2 \exp\bigl(j2\pi f_k(\tau + 1) + j2\pi\gamma_k t_n\bigr) \\ &\quad + \sum_{i=1}^{K-1}\sum_{j=i+1}^{K} \Gamma\bigl[R_{s_i s_j}^C(t,\tau) + R_{s_j s_i}^C(t,\tau)\bigr] \\ &\quad + \sum_{k=1}^{K} \Gamma\bigl[R_{s_k v}^C(t,\tau) + R_{v s_k}^C(t,\tau)\bigr] \\ &\quad + \Gamma\bigl[R_v^C(t,\tau)\bigr]. \end{aligned} \tag{4}$$

Then we take FTs of (4) in terms of τ and t_n to obtain the LVD as

$$\begin{aligned} L_x(f,\gamma) &= F_\tau\bigl\{F_{t_n}\bigl\{\Gamma\bigl[R_x^C(t,\tau)\bigr]\bigr\}\bigr\} = \sum_{k=1}^{K} L_{s_k}(f,\gamma) \\ &\quad + \sum_{i=1}^{K-1}\sum_{j=i+1}^{K} \bigl(L_{s_i s_j}(f,\gamma) + L_{s_j s_i}(f,\gamma)\bigr) \\ &\quad + \sum_{k=1}^{K} \bigl(L_{s_k v}(f,\gamma) + L_{v s_k}(f,\gamma)\bigr) + L_v(f,\gamma), \end{aligned} \tag{5}$$

where $F\{\cdot\}$ means the FT and its subscript indicates the variable associated with the operation. The auto-term $L_{s_k}(f,\gamma)$ of the kth signal component is expressed as

$$L_{s_k}(f,\gamma) = A_k^2 e^{j2\pi f}\delta(f - f_k)\delta(\gamma - \gamma_k). \tag{6}$$

The second term of (5) is associated to the cross-terms among signal components and the third term is the cross-terms among signal components and noise. The LVD has a property of asymptotic linearity that results in very small cross-terms among different components comparing to the auto-terms in (6). If $v(t) = 0$ in (1), we therefore have

$$L_s(f,\gamma) \approx \sum_{k=1}^{K} L_{s_k}(f,\gamma). \tag{7}$$

From the LVD domain, all the parameters of LFM components, i.e., centroid frequencies and chirp rates, are obtained by the peak locations (f_k, γ_k) of $L_x(f,\gamma)$ in (5). For a signal containing three LFM components, Fig. 1 shows three sharp peaks to represent the signal components and very small values that are related to the cross-terms.

3. Performance analysis

This section compares performances of the LVD in terms of signal distribution concentration in the FCR, detection probability of LFM signals, representation errors and computational complexity with those achieved by the FrFT [12] and the 2nd order PFT [10,11], which have been reported to be suitable for revealing the frequencies and chirp rates of multiple LFM components. In this paper, we assume that the dimension sizes of representing the frequency and chirp rate by the PFT and the number of angles used