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Performance analysis of a discrete-time $Geo/G/1$ queue with randomized vacations and at most J vacations ☆

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ABSTRACT

This paper presents the analysis of a discrete-time $Geo/G/1$ queue with randomized vacations. Using the probability decomposition theory and renewal process, two variants on this model, namely the late arrival system with delayed access (LAS-DA) and early arrival system (EAS), have been examined. For both the cases, recursive solution for queue length distributions at arbitrary, just before a potential arrival, pre-arrival, immediately after potential departure, and outside observer's observation epochs are obtained. Further, various performance measures such as potential blocking probability, turned-on period, turned-off period, vacation period, expected length of the turned-on circle period, average queue length and sojourn time, etc. have been presented. It is hoped that the results obtained in this paper may provide useful information to designers of telecommunication systems, practitioners, and others.

1. Introduction

Parallel to continuous-time queues, discrete-time queues are more suitable to model and analyze the digital communication system, and thus it has gained importance due to its applicability in the performance analysis of telecommunication systems. Classical examples are synchronous communication systems (slotted ALOHA) or packet switching systems with time slotting. Currently, ATM multiplexer in the broadband integrated services digital network (B-ISDN), circuit-switched time-division multiple access (TDMA) systems and slotted carrier-sense multiple access (CSMA) protocols have become a powerful incentive for the study of discrete-time queueing systems. A detailed discussion and its applications can be found on the books by Bruneel and Kim [1] and Woodward [2].

Queueing systems with server vacations have been well studied by numerous researcher since Levy and Yechiali [3]. The study on vacation queues has been applied to many practical systems, such as computer networks, communication systems and production management and so on. See details in the surveys of Doshi [4]. Keilson and Servi [5] introduced another type of variant vacation — Bernoulli vacations in which, after each service completion, the server takes a vacation with probability p, or continues its busy period (if there are customers in the system) with probability $1 - p$. Following the work of Keilson and Servi, much further research on queueing system with Bernoulli vacations was done in [6–9]. Recently, Tadj and Choudhury et al. [10] designed an optimal management policy for a quorum queueing system with a random setup time

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under N-policy and with Bernoulli vacations. Zhang and Tian [11] investigated a Geo/G/1 queue with multiple adaptive vacations in which the number of vacation is considered as a random variable.

Ke and Chu [12] introduce a vacation discipline in which the server takes at most J vacations repeatedly until at least one customer is found waiting in the queue when the server returns from a vacation. Ke and Huang et al. [13] discuss on a continuous-time queue $M[x]$/G/1 under the vacation discipline and N-policy. In the subsequent research [14–16], Ke and Huang extend the vacation discipline to the randomized vacation policy with at most J vacations in which the server takes another vacation with probability p or remains dormant within the system with probability $1-p$ if no customer is found waiting in the queue when the server returns from a vacation, otherwise, the server starts to serve the customers immediately if there are some customers waiting for service at the end of a vacation. This pattern does not terminate until the server has taken J successive vacations. Wang, [17] and Tsung-Yin Wang, Jau-Chuan Ke et al. [18] introduce the randomized-vacation policy to the discrete-time $Geo/G/1$ queue.

Such a modified vacation discipline has potentially applications in practical systems [15], e.g., in some stochastic production and inventory control systems, such as Production-to-Order. It is assumed that customer orders for this product arrive according to a compound Poisson process. Whenever all orders are completed and no new orders arrive, the production will be stopped and the facility is performed a close-down task before closing (can be referred to an essential vacation). After the production facility is completely closed, it may be available to perform some optional jobs. The optional jobs can be referred to as other second tasks or a sequence of finite maintenances. Upon completion of each optional job, the manager checks the orders and decides whether or not to resume the major production. If at this moment the major orders are empty, a decision may be made for taking other optional jobs to be performed. This modified vacation discipline is a good approximation of such a production system.

This is a continuation of work by Wang and Ke et al. [18], it may be remarked here that, Wang and Ke et al. [18] have analyzed this queue with LAS-DA (late arrival system with delayed access) only. They use supplementary variable and Markov chain techniques to discuss the queue length distribution. However, they only obtain the probability generating function (P.G.F.) of the steady-state queue length at epoch n^+ on the aspect of queue length distribution. The new contribution of this paper to the discrete-time queue $Geo/G/1$ with randomized vacation and at most J vacations is expressed as follows. First, we discuss both LAS-DA and EAS (early arrival system) models by using a new method developed by Luo and Xiang et al. [19] which is different from the supplementary variable and Markov chain techniques. Second, we derive the recursive expressions for the steady-state queue-length distributions at just before a potential arrival (n^-), pre-arrival, arbitrary (n), immediately after potential departure (n^+) and outside observer's observation epochs, respectively. The advantage of the recursive solution is that the recursive formulae given by this paper can be used to calculate the accurate numerical value of queue length distribution, which is important in application such as Supply Chain (see [20]). Moreover, it is easy to program and requires less memory. As far as we know, the recursive solutions for queue length distributions at different epochs are new. Third, by using the new approach, we also obtain various performance measures such as potential blocking probability, turned-on period, turned-off period, vacation period, expected length of the turned-on circle period, average queue length and sojourn time, etc.

The remainder of this paper is organized as follows. Section 2 presents a description of the model. In Section 3, we discuss the LAS-DA model. The probability decomposition theory and renewal process theory are employed to analyze the both transient-state and steady-state queue length distributions at different time epochs. Various performance measures are also discussed in this Section. From the results given in Section 3, we present the analysis of EAS model in Section 4. Section 5 gains the numerical aspects to the model. Section 6 establishes a state-dependent cost function and explains the strategy used to find out the optimal joint thresholds that yield the minimum cost and gains optimal result in an given example. Conclusions are given in Section 7.

2. Model description

We consider a discrete-time single server queue with randomized vacations where the time axis is divided into equal intervals called slots and all queueing activities occur at the slot boundaries. Let the time axis be marked by $0, 1, \ldots, n, \ldots$. It is assumed that the inter-arrival times τ are independent and geometrically distributed as $P\{\tau = j\} = \lambda(1-\lambda)^{j-1}, 0 < \lambda < 1, j \geqslant 1$. Customers are served according to a first-come-first-served (FCFS) discipline. The service times for customers χ are independently and identically distributed (i.i.d.) random variables with common probability mass function (p.m.f.) $g_j = P\{\chi = j\}, j \geqslant 1$, probability generating function (P.G.F.) $G(z) = \sum_{j=1}^{\infty} g_j z^j$ and mean service time $E[\chi] = \alpha$. After all the customers are served in the queue exhaustively, the server operates a randomized vacation policy with at most J vacations. As soon as the system becomes empty, the server immediately takes a vacation (measured by slots), denoted by V, with p.m.f. $v_j = P\{V = j\}, j \geqslant 1$ and P.G.F. $v(z)$. If no customers are waiting in the system upon returning from the vacation, the server takes another vacation with probability θ or remains dormant within the system with probability $\bar{\theta} = 1 - \theta$. Otherwise, the server starts to serve the customers immediately if there are some customers waiting for service at the end of a vacation. This pattern does not terminate until the server has taken J successive vacations. If the system remains empty at the end of the Jth vacation, the server keeps idle in the system until a next arrival occurs, which evokes immediately service for the arrival during server idle period. Furthermore, various stochastic processes involved in the system are assumed to be mutually independent. We mark the model with $Geo/G/1(ES, RV)$ in this paper.