Reliability Engineering and System Safety ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Repairable system analysis in presence of covariates and random effects

M. Giorgio [a], M. Guida [b,c], G. Pulcini [c,*]

[a] Department of Industrial and Information Engineering, Second University of Naples, Aversa (NA), Italy
[b] Department of Information Engineering, Electrical Engineering, and Applied Mathematics, University of Salerno, Fisciano (SA), Italy
[c] Istituto Motori, National Research Council – CNR, Naples, Italy

ARTICLE INFO

Keywords:
Repairable systems
Power-law process
Heterogeneity
Bayes inference
Powertrain systems

ABSTRACT

This paper aims to model the failure pattern of repairable systems in presence of explained and unexplained heterogeneity. The failure pattern of each system is described by a Power Law Process. Part of the heterogeneity among the patterns is explained through the use of a covariate, and the residual unexplained heterogeneity (random effects) is modeled via a joint probability distribution on the PLP parameters. The proposed approach is applied to a real set of failure time data of powertrain systems mounted on 33 buses employed in urban and suburban routes. Moreover, the joint probability distribution on the PLP parameters estimated from the data is used as an informative prior to make Bayesian inference on the future failure process of a generic system belonging to the same population and employed in an urban or suburban route under randomly chosen working conditions.

1. Introduction

Most of the papers on the reliability analysis of repairable systems deal with the homogeneous case although homogeneous populations and homogeneous operating conditions can be hardly found in practice [1]. Indeed, in many cases a substantial heterogeneity is observed among the failure patterns of apparently identical repairable systems, due to the presence of differences among units characteristics and/or differences in operating and environmental conditions. Part of such heterogeneity can be frequently explained by the presence of assignable causes (often called explanatory variables or *covariates*). The residual part (often called the *unobserved heterogeneity* or *random effects*) remains rather unexplained, since it is caused by unassignable (unidentified or not recorded) factors. Refs. [1–7] considered, among others, heterogeneity in non-homogeneous Poisson processes, whereas Lindqvist et al. [8] discussed heterogeneity in the more general case of trend-renewal processes.

An interesting example of the joint presence of explained heterogeneity and random effects is provided by the powertrain system of a fleet of buses employed in urban and suburban routes of Naples, Italy. Routes differ among themselves mainly in the traffic conditions: suburban service is characterized by noticeable thinner traffic, with a more regular use of the engine and a less frequent use of the gearbox with respect to the urban service, so that service type is expected to have some significant effect on the powertrain reliability. However, the large variability observed in the failure patterns (depicted in Figs. 1 and 2, for urban and suburban service, respectively) is only partially explained by service type. Other factors which potentially affect the reliability of the system (inherent characteristics of each system and/or not recorded unit specific service characteristics, such as the mean slope of the roads, the mean payload, etc.) leave unexplained part of the existent heterogeneity.

This paper introduces and studies a method to analyze the failure pattern of the powertrain systems in presence of identified and unidentified sources of heterogeneity. In particular, the failure pattern of each individual system is described by a Power Law Process (PLP) [9,10], that is, the non-homogeneous Poisson process with failure intensity

$$\lambda(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}, \quad \alpha, \beta > 0 \tag{1}$$

The heterogeneity among the patterns is then modeled through the combined use of:

(a) a joint probability distribution $g(\alpha,\beta)$ on the PLP parameters α and β that models the random effects, and
(b) a dummy covariate x, that defines service type (urban or suburban service) and acts only on the distribution of the scale parameter α, via the covariate parameter q.

* Corresponding author. Tel.: +39 081 7177113; fax: +39 081 2396097.
E-mail address: g.pulcini@im.cnr.it (G. Pulcini).

http://dx.doi.org/10.1016/j.ress.2014.04.009
0951-8320/

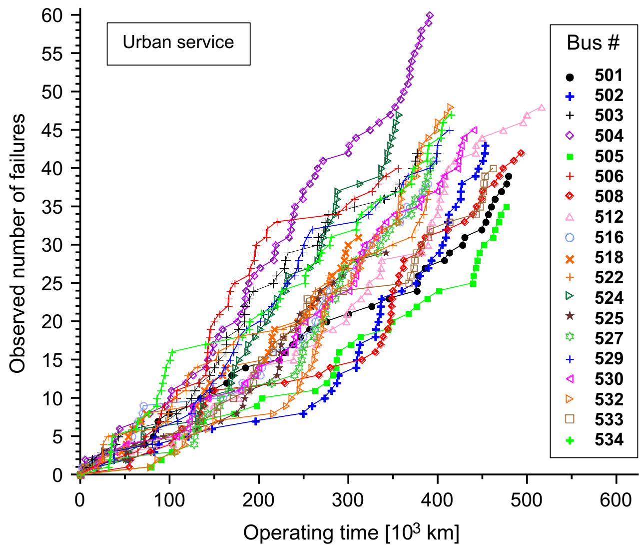

Fig. 1. The (interpolated) paths of the powertrain systems of buses employed in urban service.

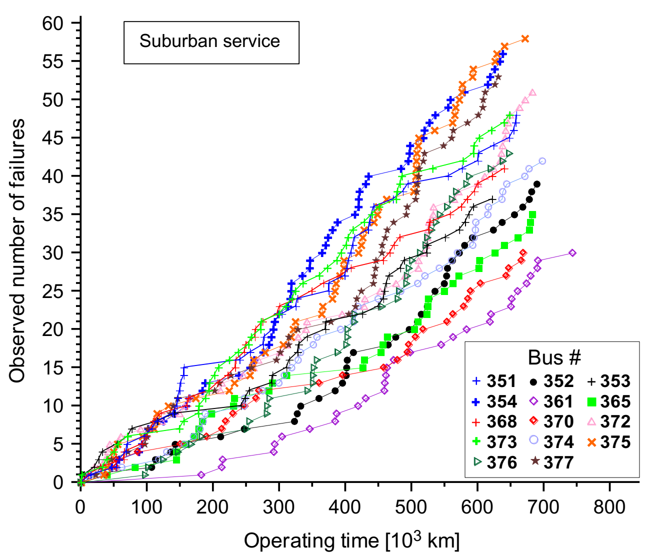

Fig. 2. The (interpolated) paths of the powertrain systems of buses employed in suburban service.

The proposed joint distribution $g(\alpha,\beta)$ is indexed by five hyperparameters $\boldsymbol{\theta}=(a,b, c,d,y)$ in order to allow the first two marginal moments of α and β and the cross moment $E\{\alpha\beta\}$ to be assessed each independently of the others.

The hyperparameters $\boldsymbol{\theta}$ and the covariate parameter q, are estimated via two different maximum likelihood procedures: (a) the herein-called 2-step procedure, which first estimates the PLP parameters α and β of each bus and then estimates $\boldsymbol{\theta}$ and q by treating the obtained ML estimates of α and β as they were the true (unit specific) values of the considered unknown parameters; and (b) the herein-called 1-step procedure, which estimates $\boldsymbol{\theta}$ and q directly from the observed failure data. The hypotheses of presence of explained heterogeneity and random effects against the null hypothesis of common parameters across all the systems are checked via the likelihood ratio testing procedure.

Based on the ML estimates of $\boldsymbol{\theta}$ and q, a joint distribution on the PLP parameters of a generic powertrain system belonging to the same population and employed in an urban or suburban route under randomly chosen working conditions is formulated. This informative distribution is then used in a Bayesian framework to make inference on the failure process of a future system, in conjunction with its early failure data. Finally, to make the potential users more confident in the effectiveness of the proposed approach, a comparison between the results obtained adopting the proposed informative Bayesian procedure and those obtained without using prior information closes the work.

2. Powertrain data and preliminary heterogeneity analyses

The application refers to the failure data of the powertrain system of $m=33$ buses built by IVECO and Breda Menarinibus, put into service in the early months of 1999 and observed until December 31, 2004. The main design characteristics (weight, size, maximum load capacity) of the IVECO and Breda Menarinibus buses were nearly identical. Again, all the buses mounted an identical copy of the powertrain (that includes a FIAT engine, a transmission with ZF gearbox, driveshafts, and differentials).

The $m_U=19$ Breda Menarinibus buses ($i=1,\ldots,19$) were employed in urban lines of the city of Naples, whereas the $m_S=14$ IVECO buses ($i=20,\ldots,33$) were employed in suburban lines of Naples suburbs. The suburban service (SS) is characterized by noticeably thinner traffic, with a more regular use of the engine and a less frequent use of the gearbox with respect to the urban service (US), and hence presence of heterogeneity between the two abovementioned groups of buses is expected.

The number n_i of observed failures ranges from 29 to 60, for a total of $n_T=\sum_{i=1}^{33} n_i=1408$ failures, and the operating times T_i of the urban and suburban buses are, on average, equal to 420,114 km and 672,434 km, respectively, for a total of 17,396,230 km covered (see Table 1).

Table 1
Failure data of powertrain systems.

i	Bus #	Type of service	n_i	T_i [km]	Failure/time truncated
1	501	Urban	39	486,997	Time
2	502	Urban	43	453,145	Time
3	503	Urban	42	376,328	Failure
4	504	Urban	60	394,191	Time
5	505	Urban	35	485,743	Time
6	506	Urban	40	356,119	Time
7	508	Urban	42	493,571	Time
8	512	Urban	48	521,180	Time
9	516	Urban	38	388,890	Failure
10	518	Urban	31	330,263	Time
11	522	Urban	37	393,041	Time
12	524	Urban	47	373,716	Time
13	525	Urban	29	341,589	Failure
14	527	Urban	41	418,028	Time
15	529	Urban	45	417,077	Time
16	530	Urban	45	439,786	Failure
17	532	Urban	48	424,950	Time
18	533	Urban	40	461,820	Failure
19	534	Urban	47	425,723	Time
20	351	Suburban	48	599,243	Time
21	352	Suburban	39	690,295	Time
22	353	Suburban	37	637,487	Time
23	354	Suburban	56	642,521	Time
24	361	Suburban	30	742,873	Failure
25	365	Suburban	35	688,488	Time
26	368	Suburban	41	654,260	Time
27	370	Suburban	30	678,418	Time
28	372	Suburban	51	702,132	Time
29	373	Suburban	48	654,295	Time
30	374	Suburban	42	700,230	Time
31	375	Suburban	58	676,184	Time
32	376	Suburban	43	662,579	Time
33	377	Suburban	53	632,632	Time

Please cite this article as: Giorgio M, et al. Repairable system analysis in presence of covariates and random effects. Reliability Engineering and System Safety (2014), http://dx.doi.org/10.1016/j.ress.2014.04.009