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Performance analysis of Hurst exponent estimators using surrogate-data and fractional lognormal noise models: Application to breathing signals from preterm infants

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ABSTRACT

The use of the Hurst exponent (H) to quantify the fractal characteristics of biological signals and its potential to detect abnormalities has aroused, recently, the interest of many researchers. Numerous techniques to estimate H are described in the literature, yet the choice of the most performing one is not straightforward. In this paper, we proposed some tests using artificial signals from experimental data and stochastic models to evaluate the robustness of three estimation techniques. Different surrogate-data tests, including a novel method to parametrize the degree of correlation in experimental signals with H (Hurst-adjusted surrogates), were first carried out. Then, simulated signals with prescribed H were obtained from fractional Gaussian noise modified properly to follow the lognormal laws observed in empirical data. The tests were applied to examine detrended fluctuation analysis (DFA), discrete wavelet transform and least squares based on standard deviation (LSSD) methods in the particular case of inter-breath interval signals from preterm infants. Simulations showed that none of the estimators were robust for every breathing pattern (regular, erratic and periodic) and should not be applied blindly without performing the preliminary tests proposed here. The LSSD technique was the most precise in general, but DFA was more robust with highly spiked patterns.

1. Introduction

The immaturity of the cardiorespiratory control system in premature newborns is known to be the origin of bradycardia, a decay of the heart beat rate, and apnea of prematurity (AOP), the cessation of breathing for 10–20 seconds [1]. Both events, frequently inter-related, provoke a diminution of the oxygen blood concentration increasing the risk of the newborns morbidity and mortality [2]. Other causes of apnea include infection, abnormal body temperature, or neurological problems. Regardless of their origin, sighs and respiratory pauses are the responsible of the variable manner in which the infants breathe during sleep. Typically, three different patterns can be identified: (1) Regular breathing, a quiet, low variable breathing in amplitude and frequency, (2) erratic breathing, high variable breathing in amplitude and frequency including several episodes of AOP and (3) periodic breathing, the alternation of pauses lasting a few seconds followed by several rapid and shallow breaths [3]. The inter-breath interval (IBI) signals, formed by the succession of the respiratory cycle times, appear consequently as complex signals with spikes (apneas) of variable number and duration according to the pattern. Despite their random-like aspect, IBI signals exhibit long-range dependence (LRD) or fractal properties [4] as other physiological signals from preterm infants, such as the heartbeat rate [5]. Since changes in the fractal structure are related to aging and disease [6], the study of LRD for clinical purposes has become an issue of growing interest.

A well-known measure of the fractal properties is the Hurst exponent (H). It can be estimated by a large number of time or frequency domain methods, but their performances can differ substantially depending on the way they are defined or the context where they are applied. The numerous comparative analyses available in the literature basically prescribe H in simulated data, then it is estimated by different methods to evaluate the error. The works of Taqqu et al. [7], and more recently Rea et al. [8], presented an extensive review of this kind of empirical study, analyzing several estimators with different lengths of time-series generated by fractional Gaussian noise (fGn) [9] and fractional autoregressive-integrated moving average (FARIMA) models [10]. Other works focusing on non-Gaussian conditions [11,12] tested the estimators with infinite variance FARIMA series and concluded

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that robustness decreases when short-range dependence (SRD) structures are present. The effect of non-stationarities on the estimated value of H ($\hat{H}$) has also aroused numerous studies in view of the fact that real data often contains local trends and shifts in the mean, which are known to produce a false detection of LRD and bias [11]. In such conditions, the detrended fluctuation analysis (DFA) [13] and a wavelet-based (DWT) estimator [14] result in good performances [15,16].

However, the models to generate simulated time-series in the mentioned works might not be realistic for some experimental data, and more specifically, IBI signals from preterm infants. In a preliminary study [17], we analyzed five estimators through a basic model which approximated apneic patterns adding spikes ad hoc to fGn, finding out that DFA, DWT and the least squares based on standard deviation method (LSSD) [18] showed the best performances. The present work not only analyzes accurately the structure of IBIs, but also formalizes more rigorous models to generate artificial signals, with the purpose of evaluating the three aforementioned methods under realistic conditions.

This paper is organized as follows: In Section 2, we first introduce long-range dependent processes, the Hurst exponent, and the three estimators under evaluation. In Section 3, we explain how the real IBI signals are obtained and analyzed with surrogate-data tests, as well as how artificial IBIs are generated. Next, Section 4 describes the characteristics of IBI signals and the error of estimations through the different tests. A discussion of the main implications of these findings follows in Section 5 and some general remarks conclude this paper.

2. Long-range dependence and the Hurst exponent

2.1. Definitions

Long-range dependence or long-range memory is a property naturally present in many physical phenomena. It is characterized by self-similar (or fractal) behavior, meaning that similar statistical properties are preserved at different scale levels, which are related by a constant known as the Hurst exponent (H) [19]. By contrast, short-range dependence exhibits statistical similarity only at short scales. LRD was firstly reported in hydrology and subsequently used in diverse areas of research such as geophysics, econometrics, network traffic and biology. Among the several definitions of LRD, the approach based on the second-order properties of a stochastic process is the most commonly accepted in the literature.

Let X_i be a stationary stochastic process with $i = 1, 2, \ldots$ the discrete sequence of observations. Let μ, $\gamma(k)$, $\rho(k)$ ($k \geqslant 0$) denote the mean, the auto-covariance and the autocorrelation, and $\sigma^2 = \gamma(0)$ the variance of the process. X_i is then considered self-similar if it has an autocorrelation of the form:

$$\rho(k) \sim k^{-\beta} L(i), \quad \text{as } k \to \infty, \tag{1}$$

where $\beta \in [0, 1]$, and L is a slowly varying function at infinity, i.e.

$$\lim_{i \to \infty} \frac{L(ix)}{L(i)} = 1 \quad \text{for any } x > 0.$$

Given m a positive scalar representing a scale greater than 1 ($X_k^{(1)} = X$), a new data-series, the mean aggregated stochastic process, is obtained for every scale:

$$X_k^{(m)} = \frac{1}{m} \sum_{i=km}^{(k+1)m-1} X_i, \tag{2}$$

with an autocorrelation denoted $\rho^{(m)}(k)$ at each scale m.

The process X is called second-order (exactly) self-similar with a characteristic Hurst exponent ($H = 1 - \beta/2$) if the variance and autocorrelation are the same at each scale, so for all $m = 1, 2, 3, \ldots,$

$$\operatorname{var}\left[X^{(m)}\right] = \sigma^2 m^{2H-2}, \tag{3}$$

$$\rho^{(m)}(k) = \rho(k) = \frac{1}{2}\left[(k+1)^{2H} + (k-1)^{2H}\right] - k^{2H}. \tag{4}$$

In finite processes (with k large enough), X is defined asymptotically self-similar if $\rho^{(m)}(k) \to \rho(k)$ as $m \to \infty$. Self-similar processes are then scale-invariant, i.e. autocorrelations at low scale levels (aggregate processes $X_k^{(m)}$) are similar at higher scale (X).

The long-range dependence property is given by the fact that autocorrelation is non-summable, $\sum_k \rho(k) = \infty$, and described by a slow, positive decaying function. The Hurst exponent ranges from 0 to 1 but LRD, which describes natural phenomena, implies $H > 0.5$. A value equal to 0.5 means that the process is completely uncorrelated or random (white noise) and values under 0.5 describe processes with negative correlations or with an anti-persistent behavior.

2.2. H estimators under analysis

A considerable number of methods to estimate H, providing different approaches to quantify the self-similar behavior, have been proposed and analyzed in the literature. In this work, we considered the detrended fluctuation analysis and the discrete wavelet transform-based, already employed in respiratory signals, and the least squares based on standard deviation, a method used in hydrology yet unexplored in biomedical data.

2.2.1. Detrended fluctuation analysis

DFA, introduced by Peng et al. [13] to estimate long-range dependence in non-stationary signals, has already been used to quantify the fractal content in IBI series from adults [20] and infants [21]. The data-series X_n of length N are first integrated and then divided into blocks of equal size m. A least squares line, representing the local trend, is fit to the data in each block. The y coordinate of the fitted line, $y_m(k)$, is then subtracted to the integrated series $y(k)$ to remove the trend in each box. Next, $F(m)$, the root-mean-square fluctuation of this integrated and detrended time-series is calculated:

$$F(m) = \sqrt{\frac{1}{N} \sum_{k=1}^{N} \left[y(k) - y_m(k)\right]^2}. \tag{5}$$

This function is repeated successively over all time scales (box sizes) to characterize its relationship with the box size m. A power law, given by $F(m) \sim m^{\alpha}$ as $m \to \infty$, indicates the presence of LRD. α is the scaling exponent, a generalization of the Hurst exponent, and it is obtained by finding the slope of the line relating $F(m)$ to m in a log plot.

2.2.2. Discrete wavelet transform-based estimation method

Abry and Veitch [14] proposed a semi-parametric joint estimator of H based on the DWT, probed to be robust with non-stationarities even when signals contains SRD. It takes advantage of the scaling properties of the wavelet basis, which captures optimally the scaling self-similar nature of LRD processes.

Briefly, the method performs first a wavelet decomposition of a given discrete time-series X_n, providing $d_x(j,k)$, the wavelet coefficients or details. Next, at each fixed octave j the details are squared and then averaged across k to produce an estimate of the variance of the wavelet coefficients, called μ_j. A plot of $\log_2(\mu_j)$ against j is done to identify the range of octaves where scaling occurs. Finally, H is computed by performing a weighted linear regression over those scales. The algorithm employed in the present