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Performance analysis of an $M/G/1$ queueing system under Bernoulli vacation schedules with server setup and close down periods

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ABSTRACT

This paper is concerned with the analysis of a single server queueing system subject to Bernoulli vacation schedules with server setup and close down periods. An explicit expression for the probability generating function of the number of customers present in the system is obtained by using imbedded Markov chain technique. The steady state probabilities of no customer in the system at the end of vacation termination epoch and a service completion epoch are derived. The mean number of customers served during a service period and the mean number of customers in the system at an arbitrary epoch are investigated under steady state. Further, the Laplace-Stieltjes transform of the waiting time distribution and its corresponding mean are studied. Numerical results are provided to illustrate the effect of system parameters on the performance measures.

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1. Introduction

Queueing systems with server vacations have been widely used to investigate the behavior of different performance measures in many practical systems. A vacation in a queueing system is a period of time where the server may not be fully available. During the vacation periods, the server may also perform other supplementary tasks. Over the last three decades, queueing systems with vacations have been investigated extensively because of their applications in modeling the digital data distributed switching systems, modern telecommunication systems, wireless ad hoc networks, computer communication networks and manufacturing/production systems. Readers are referred to the excellent surveys of Doshi (1986, 1990) and Bischof (2001), and monographs of Takagi (1991), Conti, Gregori, and Lenzini (1997), Giambene (2005) and Tian and Zhang (2006).

The performance of such a system may be measured by the waiting times experienced both by tasks in the primary queue and by the secondary tasks executed during the vacations. The appropriate trade-off between these two conflicting performance measures can be obtained by carefully designing the processor's service schedule or discipline, i.e., establish rules that specify when the processor is allowed to take a vacation from the primary queue.

There are many possible service schedules in queueing systems such as single or multi- server involving single vacation or multiple vacations. The server may initiate a vacation at random time, after serving all the customers in the primary queue and the queue is empty (exhaustive service) or completion of exactly one customer (1-limited service) or serving at most K customers (K-limited service) in the primary queue. Also, based on the applications, when the server finishes a vacation and there is no customer to be served in the queue, the server may take another vacation (multiple vacation system) or it may wait, ready to serve until a new customer arrives (single vacation system). The queueing systems with single or multiple vacation have been investigated by Levy and Yechiali (1975, 1976).

Another important vacation queueing model is Bernoulli vacation scheduling service (see Keilson & Servi (1986)). In this service discipline, when the server visits a queue, at least one customer, if any, is served. After the completion of its service, the server switches to the next vacation if there are no customers. If customers remain, however, in the queue, the next customer is served with probability $p(0 \leqslant p \leqslant 1)$ and the server repeats this procedure or the server switches to the next vacation with probability $1 - p$. The important merit of the Bernoulli scheduling is the existence of a control parameter p.

Servi (1986) has presented an approximate procedure to calculate the average waiting time of an asymmetric cyclic service token ring system under the Bernoulli scheduling service disciplines. Kella (1989) has suggested a general Bernoulli scheme according to which a single server goes on K consecutive vacations with

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probability p_K if the queue upon its return is empty. The busy period of the $M/G/1$ vacation queue with Bernoulli schedules has been studied by Ramaswamy and Servi (1988). Tedijanto (1990) has investigated the stochastic behavior of a polling system under the Bernoulli vacation scheduling service discipline in detail and has computed the average waiting time of a symmetric system. Further, Ghafir and Silio (1993) have discussed the Bernoulli vacation schedule for predicting waiting times on PLAYTHROUGH rings whose stations use a FIFO discipline. Choudhury, Tadj, and Paul (2007) have discussed the steady state analysis of a batch arrival queue with two phases heterogenous service and a setup time under Bernoulli vacation schedule. It is also assumed that after two successive phase services or first vacation, the server goes for multiple vacations until the server finds a new batch of customers in the system. Using the imbedded Markov chain technique, the stationary queue size distribution, the busy period distribution and some system performance measures have been investigated under steady state. Later, an $M^X/G/1$ queue with Bernoulli vacation schedule under random setup time under a restricted policy is considered by Choudhury (2008). In this queueing system the server turn off when the system becomes empty and turns on only when at least one customer arrives at the system. It is also assumed that during busy periods and vacations periods restricted admission policy is adopted, i.e., not all batches of customers are allowed to join the system. For this system, the probability generating function of the system size and the corresponding mean system size at a random epoch are obtained. The stochastic decomposition theorem (see Fuhrmann & Cooper (1985)) for the system size is also studied. Moreover, Choudhury and Tadj (2011) have studied the batch arrival queueing systems with two phases of heterogeneous service under Bernoulli vacation schedule. Recently, Krishna Kumar and Pavai Madheswari (2009) have analyzed a multi-server queue in which each server takes vacation according to Bernoulli vacation scheduling discipline. For this system, the stationary queue length distribution, the system busy period and waiting time distribution have been studied using the matrix geometric solution technique.

Many queueing situations containing "mechanical parts" need a setup period to serve the customers. The server setup period corresponding to the preparatory work of the server starts before the service period. Examining queueing systems which combine the N-policy with a setup period, Baker (1973) has first proposed the N-policy $M/M/1$ queueing system with exponential setup period. Borthakur, Medhi, and Gohain (1987) have extended Baker's results to the general setup period and obtained the steady state results of the the system performance measures under the N-policy. Various queueing systems with setup period have been investigated by several researchers such as Minh (1988), Medhi and Templetion (1992), Takagi (1993), Lee and Park (1997), Hur and Paik (1999) and Artalejo, Economou, and Lopez-Herrero (2005).

There have been several models developed in recent years for vacation queueing systems with close down period. The concept of close down period was first introduced by Takagi (1991). Niu, Takahashi, and Endo (1998, 2003) and Niu and Takahashi (1999) have discussed the performance measures of the switched virtual channel connection (SVCC) by a close down period, which is corresponding to an inactive timer during which the SVCC resource is reserved to anticipate more customers (packets) from the same internet protocol (IP) flow. Hassan and Atiquzzaman (1997) and Sakai, Takahashi, Takahashi, and Hasegawa (1998) studied non-Markovian queues with setup and close down periods and their applications to SVCC based ATM networks.

Later, Ke (2007) has investigated the operating characteristics of a batch arrival queueing system under modified vacation policy with setup and close down periods and its applications to the production/inventory systems. Recently, Baek and Choi (2011) have determined the average message delay and the average power consumption of a mobile station by using a queueing system with close down period.

In the literature, the steady state performance measures are discussed for various exhaustive service queueing systems with multiple vacations and server's setup/close down periods. However, as far as we are aware, there is no analysis of queueing systems with Bernoulli vacation scheduling discipline and setup/fixed length of close down periods. This motivated us to develop an analysis of a single server queueing system with above mentioned features. This article is organized in the following manner: In Section 2, the mathematical model is described. An explicit expression for the probability generating function of the number of customers present in the system is obtained by using imbedded Markov chain in Section 3. Some key performance measures of the system under steady state conditions are also derived. In Section 4, the Laplace-Stieltjes transform of the waiting time distribution and its corresponding mean are determined. We deduce some special cases of our queueing system which are consistent with existing results in the literature. Some numerical examples are presented to illustrate the effect of system parameters on the performance measures in Section 5. Finally, we draw some conclusions in Section 6.

2. Model description and analysis

We consider a single-server queue with Bernoulli vacation schedules, setup/startup and close down times. The arrival process of customers is Poisson with rate λ. Arriving customers form a single waiting line based on the order of their arrivals and are served according to first-come first-served (FCFS) service discipline. The server can serve only one customer at a time. Let S_n be the service time duration of the nth customer. The service time durations $\{S_n; n = 1, 2, 3, \ldots\}$ be independent and identically distributed (i.i.d) random variables with a cumulative distribution function $F_S(t)$, the corresponding probability density function (p.d.f) $f_S(t)$, Laplace-Stieltjes Transform (LST) $F_S^*(s)$ and finite first and second moments $E(S)$ and $E(S^2)$. The server takes vacations according to Bernoulli vacation schedules as proposed by Keilson and Servi (1986), i.e., after each service completion epoch, if the queue is non-empty, either the server serves the next customer with probability $p(0 \leqslant p \leqslant 1)$ or the server takes a vacation with probability $1-p$. Let V_n be the length of the nth vacation time duration of the server. The vacation time lengths$\{V_n; n = 1, 2, 3, \ldots\}$ be i.i.d random variables with a cumulative distribution function $F_V(t)$, the corresponding p.d.f $f_V(t)$, LST $F_V^*(s)$ and finite first and second moments $E(V)$ and $E(V^2)$. Once the server finishes the service of all the customers and finds the system is empty, i.e., at the end of each busy period, the system enters into a close down period of a fixed length Δ (a time out period of Δat the end of a busy period). During the close down period/time out period, if any new customer arrives, the server resumes service (at the end of the close down period) so that the system busy period begins again. On the other hand, if the close down period expires before the arrival of a customer, the server immediately goes for multiple vacations, i.e., the server takes vacations repeatedly until it finds at least one customer queued for the service upon returning from a vacation. Further, it is assumed that if the server returns from a vacation to find the system not empty, it immediately reactivates and performs a setup/startup time with random length before starting its service. Let U_n be the length of setup time which might be encountered at the beginning of the nth cycle. The setup time periods $\{U_n; n = 1, 2, 3, \ldots\}$ be i.i.d random variables with a cumulative distribution function $F_U(t)$, the corresponding p.d.f $f_U(t)$, LST $F_U^*(s)$ and finite first and second moments $E(U)$ and $E(U^2)$. As soon as the server

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