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Design and performance analysis of a nanogyroscope based on electrostatic actuation and capacitive sensing

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ABSTRACT

In this paper, a vibrating beam gyroscope with high operational frequencies at mode-matched condition is proposed. The model comprises a micro-cantilever with attached tip mass operating in the flextural–flextural mode. The drive mode is actuated via the electrostatic force, and due to the angular rotation of the base about the longitudinal axis. The secondary sub-nanometric vibration is induced in the sense direction which causes a capacitive change in the sense electrodes. The coupled electro-mechanical equation of motion is derived using the extended Hamilton's principle, and it is solved by direct numerical integration method. The gyroscope performance is investigated through the simulation results, where the device dynamic response, rate sensitivity, resolution, bandwidth, dynamic range, *g* sensitivity and shock resistance are studied. The obtained results show that the proposed device may have better performance compared to commercial micro electromechanical gyroscope characteristics.

1. Introduction

Gyroscopes as angular rate sensors find broad range of applications from automotive to aerospace and consumer electronics industries. Optical and mechanical gyroscopes in comparison with micro-machined vibratory gyroscopes are more accurate and have better scale factor stability [1]. However, they are too expensive and too bulky. The micro-machined gyroscopes due to their small size, light weight, low power consumption and low cost find application in automotive and electronic consumer and makes them ideal for use in handheld applications. Today, common silicon micro-machined vibratory gyroscopes operate in low frequency range (3–30 kHz) which suffer from the low bandwidth and bias stability and low reliability due to low shock resistance [2]. To increase the scale factor, resolution, and bias stability a high quality factor is desirable which is accomplished by device packaging in high vacuum. It may be further noted that, even in high vacuum, the effective quality factor is restricted by thermoelastic damping [3,4]. In some applications such as automotive, a very fast response time is required. Consequently, higher bandwidth is desired which could be achieved by increasing the resonant frequency or decreasing the quality factor of the gyroscope. Although, the later, decreases the scale factor, and the performance of the device.

Micro/nano cantilevers have fined various applications such as mass sensor [5,6], resonators [7], atomic force microscopes [8] and vibrating beam gyroscopes. Electrostatic actuation as a versatile and applicable method is employed to design and develop various micro/nano electromechanical systems (MEMS/NEMS) [9–13].

Recently vibrating beam gyroscopes as an interesting issue has obtained more attention. The optimal size and the level of thermal noise of a vibrating beam gyroscope have been investigated and it is shown the longer the beam the lower the

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Nomenclature

a	Tip mass width
A_b	Cross sectional area of the beam
A_M	Cross sectional area of the tip mass
A_s, A_d	Sense and drive electrode area
b	Beam width
BW	Bandwidth
c_d, c_s	Effective damping coefficient in the drive and sense directions, respectively
c_{SL}, c_{SQ}	Damping coefficient for sliding and squeeze film damping, respectively
C_P	Specific heat under constant pressure
C_{s1}, C_{s2}	Sense capacitances
E	Young's modulus
f_n	Resonant frequency
F_d	Electrostatic drive force
g_d, g_s	Drive and sense gaps, respectively
H_l	Heaviside function
i_s	Total sense current
$I(x)$	Area moment of inertia function
I_b, I_M	Area moment of inertia of the beam and tip mass, respectively
k_B	Boltzmann constant
K_n	the Knudsen number
l	Total beam length
l_M	Length of the tip mass
l_1	$l-l_M$
l_2	l
$m(x)$	Mass per unit length function
M	Effective mass
P	Pressure
q_0	Equilibrium charge
$q_1(t)$, $q_2(t)$	Excess charges in sense capacitors
Q	Quality factor
$\mathbf{r}$	Position vector
R	Feedback resistor
t	Time
T	Kinetic energy
T_0	Temperature
$v(x,t)$	Deflections along the moving Cartesian coordinate y
v_{max}	Maximum beam deflection
V	Mechanical potential energy
V_o	Output voltage
V_s, V_d	Sense and drive voltage, respectively
$w(x,t)$	Deflections along the moving Cartesian coordinate z
W_e	Electrical potential energy
W_{nc}	Non-conservative work
x,y,z	Moving Cartesian coordinate
X,Y,Z	Inertial reference frame
α	Thermal expansion coefficient
ε	Vacuum permittivity
κ	Thermal diffusivity
λ	Gas mean free path
μ	Gas viscosity
μ_{SL}, μ_{SQ}	Effective viscosity of the gas in sliding and squeeze film damping, respectively
$\phi_i(x)$	ith linear undamped mode shape
ρ_b, ρ_M	Volume density of the beam and tip mass, respectively
ζ	Damping ratios
τ_z	Relaxation time
ω	Angular frequency of excitation
ω_n	Natural frequency
Ω	Base angular velocity
Ω_{min}	Gyroscope resolution

Subscripts

d	Drive
EFF	Effective
s	Sense
SL	Sliding
SQ	Squeeze
TH	Thermal

thermal noise [14]. Rotating beams with piezoelectric films as an angular rate sensor has been investigated [15]. The Finite element modeling of a rotational motion sensor which uses tuning fork to sense the angular rate has been performed and dynamic properties of the sensor has been investigated [16]. Dynamic characteristics of a gyroscope based on beam structure have been investigated in which the bandwidth and sensitivity analysis have been performed [17]. Vibrating beam microgyroscope under general base motion has been studied and comprehensive dynamic model has been presented [18]. Silicon angular rate sensor for automotive applications has been designed and simulated; the sensor was based on a tuning fork principle and piezoelectric excitation [19]. Piezoelectrically actuated vibrating beam gyroscope operated in flexural/torsional mode has been proposed and a dynamic model has been presented. It has been shown the device performance is adversely affected by the cross-axis effects [20]. Dynamic stability of vibrating beam gyroscope subjected to rate fluctuations has been analyzed using the method of averaging, and closed-form stability conditions have been presented [21].

Several companies provide microgyroscopes as standard components. The first of these was an integrated z-axis gyroscope announced in 2002 by Analog Devices Inc. (ADI), offering a very high resolution of $0.05^\circ/\mathrm{s}/\sqrt{\mathrm{Hz}}$ [22]. Robert Bosch presented a z-axis gyroscope with $1.5^\circ/\mathrm{s}/\sqrt{\mathrm{Hz}}$ resolution and 30 Hz frequency bandwidth [23]. In 2006, a high resolution gyroscope (SiRRS01) was introduced by Silicon Sensing, with noise density of $0.35^\circ/\mathrm{s}/\sqrt{\mathrm{Hz}}$ and 50 Hz bandwidth [24]. Northrop Grumman Corporation introduced a MAG-16 MEMS gyroscope with $0.03^\circ/\mathrm{s}/\sqrt{\mathrm{Hz}}$ resolution and operational range of 150°/s [25]. Later on, in 2007, InvenSense Inc. offered the first commercialized dual-axis integrated gyroscope (IDG-300). Its resolution was claimed of $0.014^\circ/\mathrm{s}/\sqrt{\mathrm{Hz}}$ in operational range of 50°/s [26].