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Performance analysis of the selective coefficient update NLMS algorithm in an undermodeling situation

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ABSTRACT

The selective coefficient update normalized least mean-square (SCU-NLMS) algorithm was proposed to reduce computational complexity while preserving close performance to the full-update NLMS algorithm, which brought it a lot of attention. In practical applications, the length of the unknown system impulse response is not known and, therefore, the length of the adaptive filter can be less than that of the unknown system particularly in situations when the unknown system impulse response is long. In all existing analysis of the SCU-NLMS algorithm, exact modeling of the unknown system is assumed, i.e., the length of the adaptive filter is equal to that of the unknown system impulse response. In this paper, we present mean-square performance analysis for the SCU-NLMS algorithm in an undermodeling situation and assuming independent and identically distributed (i.i.d.) input signals. The analysis model takes into account order statistics employed in the SCU-NLMS algorithm leading to accurate transient and steady state theoretical results. Analysis extends easily to the exact modeling case where expressions quantifying the algorithm mean-square performance are presented and shown to be more accurate than the ones reported in the literature. Simulation experiments validate the accuracy of the theoretical results in predicting the actual behavior of the algorithm.

1. Introduction

The selective coefficient update normalized least mean-square (SCU-NLMS) algorithm belongs to the family of adaptive algorithms that reduces computational complexity by updating a portion of the adaptive filter coefficients at each iteration. The SCU-NLMS algorithm has been a viable choice in applications where long adaptive filters are used since it was shown to maintain the closest performance among partial update NLMS algorithms to the full-update NLMS algorithm [1–7]. In all existing analysis of the SCU-NLMS algorithm [2–4,8], exact modeling of the unknown system is assumed, i.e., the length of the adaptive filter is equal to that of the unknown system impulse response. However, in practical applications, the length of the unknown system is not known, and in applications with long impulse responses, the system is normally undermodeled. Theoretical results of the SCU-NLMS algorithm with exact modeling do not necessarily apply in the undermodeling case and, therefore, there is a need to study the theoretical performance of the algorithm in an undermodeling situation.

The objective of this paper is to provide convergence analysis of the SCU-NLMS algorithm that describes its transient as well as its steady state mean and mean-square behavior in an undermodeling situation. Analysis assumes a zero-mean independent and identically distributed (i.i.d.) input signal, and will use the common independence assumption that the input signal vector is independent of the adaptive filter vector [9]. Assuming a more general input signal model, though desirable in the analysis of adaptive algorithms, makes providing explicit closed form expressions that clearly quantify the mean-square behavior of the algorithm a very difficult task due to order statistics employed by the algorithm [3,4,8]. Our analysis model takes into account order statistics thus leading to results that predict very well the actual mean and mean-square behavior of the algorithm. In [4], order statistics were applied to the analysis of the selective partial update NLMS algorithm with exact modeling, which was shown to provide more accurate results than those in [3]. Our analysis will be extended to the exact modeling case, and will result in different equations for the algorithm MSE, final excess MSE, and stability bounds than those derived in [4]. This is because authors in [4] analyze the selective partial update NLMS algorithm presented in [3] that has a slightly different coefficient update equation than the one we are analyzing here and was proposed in [1,2]. Moreover, our analysis approach is different from that in [4] where authors also assume a certain model for the white input signal. In [8], mean-square analysis was performed for the SCU-NLMS algorithm, and the effect of order statistics appears in the analysis in a simplified manner. The analytical approach used here is different from that in [8]. Simulation experiments will be conducted to verify analysis, and also comparisons are made with theoretical results from [8] for the exact modeling case.

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2. Performance analysis

Let $\mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-N+1)]^T$ be the input vector to the adaptive filter. The SCU-NLMS algorithm updates at each iteration M coefficients of the adaptive filter N coefficients that correspond to the first M maxima of $|\mathbf{x}(n)|$, where $1 \leqslant M \leqslant N$. The remaining $N-M$ coefficients are kept unchanged. Define the $N \times N$ diagonal selection matrix $\mathbf{S}(n)$, which has ones at diagonal entries indicated by $s_i(n)$, $i = N-M+1, N-M+2, \ldots, N$, that correspond to the M maxima of $|x(n-s_i(n)+1)|$, $i = 1, 2, \ldots, N$, and zeros at diagonal entries indicated by $s_i(n)$, $i = 1, 2, \ldots, N-M$, where $|x(n-s_1(n)+1)| \leqslant |x(n-s_2(n)+1)| \leqslant \cdots \leqslant |x(n-s_N(n)+1)|$. The SCU-NLMS algorithm updates its coefficients according to the following equation[1] [1,2]

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu e(n)}{\|\mathbf{x}(n)\|^2}\mathbf{S}(n)\mathbf{x}(n) \tag{1}$$

where

$$e(n) = d(n) - \mathbf{x}^T(n)\mathbf{w}(n) \tag{2}$$

is the system output error signal, $d(n)$ is the desired signal, $\mathbf{w}(n) = [w_1(n)\, w_2(n) \ldots w_N(n)]^T$ is the adaptive filter coefficient vector, and μ is the step-size. In our analysis, we assume that the desired signal is given by

$$d(n) = \mathbf{x}_L^T(n)\mathbf{w}_L^* + \eta(n) \tag{3}$$

where $\mathbf{w}_L^* = [w_1^*, w_2^*, \ldots, w_L^*]^T$ is the unknown system impulse response vector, $\mathbf{x}_L(n) = [x(n), x(n-1), \ldots, x(n-L+1)]^T$ is the input signal vector to the unknown system, where $N < L$, and $\eta(n)$ is a stationary zero-mean independent noise sequence. This model for $d(n)$ is accurate for system identification tasks such as acoustic and network echo cancellation, active noise control, and in modeling inverse-filtering tasks when the input signal is Gaussian [10]. The input signal $x(n)$ is assumed zero-mean and i.i.d. This assumption makes it possible to obtain closed form expressions that quantify the algorithm mean-square behavior and, therefore, is commonly employed in the analysis of SCU-type algorithms [3,4,8]. We further use the common independence assumption of $\mathbf{x}(n)$ and $\mathbf{w}(n)$ [9]. We write $\mathbf{x}_L(n)$ and $\mathbf{w}_L^*$, respectively, as

$$\mathbf{x}_L(n) = \begin{bmatrix} \mathbf{x}(n) \\ \bar{\mathbf{x}}(n) \end{bmatrix} \tag{4}$$

and

$$\mathbf{w}_L^* = \begin{bmatrix} \mathbf{w}^* \\ \bar{\mathbf{w}}^* \end{bmatrix} \tag{5}$$

where $\bar{\mathbf{x}}(n) = [x(n-N), x(n-N-1), \ldots, x(n-L+1)]^T$, $\mathbf{w}^* = [w_1^*, w_2^*, \ldots, w_N^*]^T$, and $\bar{\mathbf{w}}^* = [w_{N+1}^*, w_{N+2}^*, \ldots, w_L^*]^T$. Defining the $N \times 1$ coefficient error vector $\mathbf{v}(n) = [v_1(n)\, v_2(n) \ldots v_N(n)]^T$ as

$$\mathbf{v}(n) = \mathbf{w}(n) - \mathbf{w}^* \tag{6}$$

and, from Eq. (1), we can write the coefficient error update equation of the SCU-NLMS algorithm as

$$v_{s_i(n)}(n+1) = \begin{cases} v_{s_i(n)}(n) \\ \quad i = 1, 2, \ldots, N-M \\ v_{s_i(n)}(n) + \frac{\mu e(n)}{\|\mathbf{x}(n)\|^2} x(n-s_i(n)+1) \\ \quad i = N-M+1, N-M+2, \ldots, N \end{cases} \tag{7}$$

Using Eqs. (3), (4), (5), and (6), the output error in Eq. (2) can be put in the form

$$e(n) = \bar{\mathbf{x}}^T(n)\bar{\mathbf{w}}^* - \mathbf{x}^T(n)\mathbf{v}(n) + \eta(n) \tag{8}$$

and we can express Eq. (7) for $i = N-M+1, N-M+2, \ldots, N$ as

$$\begin{aligned} v_{s_i(n)}&(n+1) \\ &= v_{s_i(n)}(n) - \frac{\mu}{\|\mathbf{x}(n)\|^2} \sum_{j=1}^{N} v_j(n) x\big(n-s_i(n)+1\big) x(n-j+1) \\ &\quad + \frac{\mu}{\|\mathbf{x}(n)\|^2} \sum_{j=N+1}^{L} w_j^* x\big(n-s_i(n)+1\big) x(n-j+1) \\ &\quad + \frac{\mu}{\|\mathbf{x}(n)\|^2} x\big(n-s_i(n)+1\big)\eta(n) \end{aligned} \tag{9}$$

2.1. Mean convergence

Taking the expected value of both sides of Eq. (9), and using the independent assumption yields

$$\begin{aligned} &E\{v_{s_i(n)}(n+1)\} = [1 - \mu\beta_{2,N}\sigma_{2,s_i}]\, E\{v_{s_i(n)}(n)\} \\ &\quad i = N-M+1, N-M+2, \ldots, N \end{aligned} \tag{10}$$

where we defined the quantities

$$\beta_{p,N} = E\left\{\frac{1}{\|\mathbf{x}(n)\|^p}\right\} \tag{11}$$

and

$$\sigma_{p,s_i} = E\left\{x^p\big(n-s_i(n)+1\big)\right\} \tag{12}$$

In obtaining Eq. (10), we assume a sufficiently long adaptive filter such that the variations in $\frac{1}{\|\mathbf{x}(n)\|^2}$ are small enough to justify the approximation

$$\begin{aligned} &E\left\{\frac{x(n-s_i(n)+1)x(n-j+1)}{\|\mathbf{x}(n)\|^2}\right\} \\ &\quad \approx E\left\{\frac{1}{\|\mathbf{x}(n)\|^2}\right\} E\left\{x\big(n-s_i(n)+1\big)x(n-j+1)\right\} \end{aligned} \tag{13}$$

For a Gaussian zero-mean i.i.d. input signal, it was shown in [11] that $\beta_{2,N}$ can be computed as

$$\beta_{2,N} = \frac{\Gamma(\frac{N-2}{2})}{2\sigma_x^2 \Gamma(\frac{N}{2})}, \quad N > 2 \tag{14}$$

where $\Gamma(u) = \int_0^\infty r^{u-1} e^{-r}\, dr$ is the complete Gamma function, and σ_x^2 is the variance of the input signal. The calculation of σ_{p,s_i} is a problem of order statistics [12,13], and not every distribution has a closed form solution for σ_{p,s_i}. However, we derive in Appendix A expressions for σ_{p,s_i} that can be calculated numerically, where p is even. It should be noted that for a Gaussian i.i.d. input signal, explicit closed form expressions exist for moments of order statistics from an i.i.d. chi-square distribution with one degree of freedom [14]. Note that

[1] The selective partial update NLMS (SPU-NLMS) algorithm has the update equations $\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu e(n)}{\|\mathbf{S}(n)\mathbf{x}(n)\|^2}\mathbf{S}(n)\mathbf{x}(n)$ [3]. It is shown in [4] that the SPU-NLMS algorithm becomes the SCU-NLMS algorithm with unity step-size when the SPU-NLMS algorithm is used with the step-size that gives the fastest convergence speed of the algorithm.