Available online at www.sciencedirect.com
SCIENCE DIRECT®
Int. J. Production Economics 103 (2006) 527–534
international journal of production economics
www.elsevier.com/locate/ijpe

Optimal ordering policy in a distribution system

Jing-An Li[a], Yue Wu[b], Kin Keung Lai[a,c,*], Ke Liu[d]

[a]Department of Management Sciences, City University of Hong Kong, Hong Kong, PR China
[b]School of Management, University of Southampton, UK
[c]College of Business Administration, Hunan University, Changsha, 410082 Hunan, PR China
[d]Institute of Applied mathematics, Academy of Mathematics and System Sciences, Chinese Academy of Sciences, Beijing, PR China

Received 24 September 2003; accepted 11 November 2005
Available online 3 March 2006

Abstract

In conventional inventory management, the retailers monitor their own inventory levels and place orders at the distributor when they think it is the appropriate time to reorder. The distributor receives these orders from the retailers, prepares the product for delivery. Similarly, the distributor will place an order at the manufacturer at the appropriate time.

Generally, the order that the distributor places at the manufacturer is larger than that the retailer places at the distributor. In order to afford this large order, there should exist a long-term supply contract between the manufacturer and distributor that can guarantee a stationary supply to the distributor. This paper discusses this case, and derives the optimal stationary supply, that is, the optimal ordering policy of the distributor. Also computational results are presented.

Keywords: Inventory; Production; Order-up-to policies

1. Introduction

In conventional inventory management, the retailers monitor their own inventory levels, and when a retailer thinks that it is time to reorder, an order for a quantity of the product is placed at the distributor. The distributor receives these orders from the retailers, prepares the product for delivery. Similarly, when the distributor thinks it is time to reorder, an order for a quantity of the product is placed at the manufacturer. Even in the recent vendor managed inventory replenishment that refers to the situation in which a distributor monitors the inventory levels at its retailers and decides when and how much inventory to replenish at each retailer, the distributor should place an order at the manufacturer at the appropriate time too. Generally, the order that the distributor places at the manufacturer is larger than that the retailer places at the distributor. In order to afford this large order, there should exist a long-term supply contract between the manufacturer and distributor that can guarantee a stationary supply to the distributor. As we know, a long-term stationary supply not only benefits the distributor, but also decreases the uncertainty of the supply of the manufacturer, which will benefit the manufacturer too. Then, how many stationary supply will be optimal?

*Corresponding author. Department of Management Sciences, City University of Hong Kong, Hong Kong, PR China. Tel.: +852 2788 8563; fax: +852 2788 8560.
E-mail address: mskklai@cityu.edu.hk (K.K. Lai).

0925-5273/$ - see front matter
doi:10.1016/j.ijpe.2005.11.007

For example, the gas company in Hong Kong has two production plants, where gas is made from high quality naphtha instead of heavy fuel oil. The company has settled facilities and manpower so that she can ensure a stationary gas production to satisfy the demand through their own network. Because the demand is uncertain, e.g., high during dinning time and low otherwise, the gas consumed will be uncertain too. If the company's production fits the demand like a glove, the company may always regulate her facilities and manpower. However, this regulation may cause unknown large costs. Then, how to use the stationary production to satisfy the uncertain demand so that total cost can be minimized will be an important problem. And this case is similar to the general one described above.

In general terms, when the demand is deterministic, this problem can always be solved by using the classical economic-order-quantity (EOQ) model, where the ordering policy is to place an order for a fixed quantity of the product at each period. And there are much literature discussing this model, e.g., Zipkin, 2000, etc. Of course, more and more complex EOQ cases are presented. For example, the EOQ model with deteriorating items (Chang, 2004), the EOQ model with process reliability considerations (Tripathy et al., 2003) etc.

In recent years, more literature discusses the case where the demand is stochastic. Considering the continuous review inventory model, Archibald (1981) discusses the continuous review (s, S) policies with lost sales, and develops a method to calculate an (s, S) policy that minimizes the average stationary cost in an inventory system. Wang and Gerchak (1996) analyze the effects of variable capacity on optimal lot sizing in continuous review environments, and obtain the optimal conditions for generally distributed variable capacity and the optimal order quantities. Mohebbi and Posner (1998) derive the stationary distribution of the inventory level in a continuous-review inventory system, and Mohebbi and Posner (2002) present the stationary distribution of the on-hand inventory using a level-crossing methodology for a continuous review inventory system with lost sales, nonunit size, multiple replenishment orders outstanding, and split deliveries. Also there are many inventory books, such as Axsater (2000), Bartmann and Beckmann (1992), Zipkin (2000), Heyman and Sobel (1984), discussing the continuous review model with (r, Q) or (s, S) policies.

It is worth noting that there is more literature focusing on the period review inventory model. Considering the stochastic single product multi-period inventory model, most of existing literature focuses on the optimal (s, S) policy, which is to order up to S when the inventory level is less than s, to do noting otherwise. When there is no setup cost per any order, s will equal S and this policy is always called myopic policy.

The optimal (s_n, S_n) policies for n periods are presented by Arrow et al. (1951), and proved theoretically by Scarf (1960) using the k-convex function. For infinite horizon models with stationary data, Iglehart (1963) shows that the (s, S) policy is optimal too under the assumption of a convex holding and shortage cost function. Thenceforth, more researchers focus on the computation and generalization of the optimal (s, S) policy, for example, Johnson (1968), Federgruen and Zipkin (1984) and Zheng (1991), etc. Recently, more literature adds more restrictions to the multi-period inventory problems and studies whether the (s, S) policy is optimal or not. Federgruen and Zipkin (1986) have shown that the optimal policy to the capacitated problem is just a simple modification of the optimal base-stock policy to the uncapacitated problem in case of the setup cost $K = 0$. Chen and Lambrecht (1996) find the examples of finite horizon where the simple structure fails to hold, and they obtain that the optimal policy does exhibit a systematic pattern of what is called X–Y band: when the inventory level drops below X, order up to capacity; when the inventory level is above Y, do nothing; and when the inventory level is between X and Y, the ordering pattern is not known. Based on a concept called (C, K)-convexity, Chen (2004) shows that the X–Y band is no more than one capacity of width, and presents a linear programming to find the optimal policy.

Our paper discusses the inventory problem facing the distributor where there exists an stationary supply to satisfy its stochastic demand. Considering that there exists a long-term stationary supply, and the extra order placed by the distributor can be satisfied with higher price by the manufacturer, and the stochastic orders placed by the retailers should be satisfied at once and backlogging and shortage are not allowed, we form this problem to be a single product, multi-period inventory problem. And we analyze the cost function that contains the ordering cost, holding cost and extra shortage cost. Comparing with the aforementioned literature, the difference and contributions are as follows. First, we prove that the cost function is a convex function of the amount of the stationary supply, so that the distributor can