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Economic analysis of an n-unit parallel redundant system based on a Stackelberg game formulation

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ABSTRACT

A parallel redundancy has been researched in a great amount of literature on reliability theory. In the real circumstances, however, it is seldom observed except some systems that require much higher reliability. This is because most of the literature only look at the manufacturer's point of view. The present study carries out an economic analysis of an n-unit parallel redundant system against a single unit system based on a Stackelberg game formulation considering both the consumer's viewpoint and the manufacturer's one. It clarifies quantitatively in what situation the manufacturer can increase his profit by dealing in the parallel redundant system.

1. Introduction

Parallelizing of units in a system is an effective method for improving system reliability. Since this method is simple to analyze for a variety of systems, it is widely introduced in a great amount of literature on reliability theory (Barlow & Proschan, 1967; Osaki, 2002).

Parallel systems are, however, not so widely employed in the actual environments as a great amount of literature have treated. The authors speculate that this is because most of the literature miss the consumer's point of view and because the consumer would purchase a parallel system only when it is worth purchasing compared with a single unit system. In redundancy allocation problem (e.g. Hsieh, 2003; Kuo & Prasad, 2000; Liang & Smith, 2004), for instance, the manufacturer seeks the optimal selection of system components in series–parallel systems considering reliability, manufacturing cost and weight, but he never takes into account the consumer's behavior for the target system.

The present study makes a comparison between a single unit system and an n-unit parallel redundant system both from the consumer's viewpoint and from the manufacturer's one. In this study, we consider a situation where a manufacturer produces both single unit systems and n-unit parallel redundant systems to deal in them under a monopoly. It is postulated that the n-unit parallel redundant system consists of n identical units to that of a single unit system. It should, furthermore, be noted that since we have assumed a monopoly, the manufacturer can determine the prices of these systems (e.g. Mankiw, 2007, chap. 15). Hence we can formulate the problem within a Stackelberg game framework (Fundenburg & Tirole, 1991; Gibbons, 1992; Osborne & Rubinstein, 1994), letting the manufacturer and the consumers be the leader and the followers, respectively. Two cases are discussed according to the consumer's revenue structures: Case 1 deals with a circumstance where the consumer's revenue is proportional to the system life, while under Case 2, the consumer's revenue significantly depends on the system reliability.

The remaining part of the paper is organized as follows. Section 2 describes the proposed model, and Sections 3 and 4 formulate and analyze the model to derive the manufacture's optimal strategy in Cases 1 and 2, respectively. Numerical examples are also illustrated in Section 5. Section 6 concludes our paper and describes future problems. Detailed proofs are presented in the appendices.

2. Assumptions and notations

In this section, we make assumptions and define notations necessary for the proposed model.

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2.1. Common assumptions and notations for Cases 1 and 2

(1) The manufacturer produces two kinds of systems using identical system units; a single unit system and an n-unit parallel redundant system which consists of n identical system units ($n \geqslant 2$). The n-unit parallel redundant system is simply referred to as an n-unit system in the following.

(2) In this study, we consider a monopoly, under which the manufacturer can determine the price of products he deals in Mankiw (2007, chap. 15). Hence, the problem is formulated within a Stackelberg game framework by regarding the manufacturer and the consumers as the leader and the followers, respectively.

(3) The cost for manufacturing a system is given by the sum of the costs for a system unit and the fixed cost. The fixed cost of a single unit system and an n-unit system are, respectively, given by b and αb, where $b > 0$ and $\alpha \geqslant 1$. It is to be noted that in the case of $\alpha = 1$, the fixed cost for an n-unit parallel system agrees with that for a single unit system.

(4) A single unit system and an n-unit system are sold at a price P_1 and $P_n (\geqslant P_1)$, respectively. It is assumed in this study that P_1 is fixed and P_n is, instead, a design variable since we seek a relative value for P_n in comparison with P_1.

(5) We consider two cases focusing on the consumer's revenue structure: In Case 1, it is assumed that the consumer's revenue is proportional to the life of the system he purchases. This can be seen for various kinds of electric and/or electronic systems. In Case 2, however, the consumer's revenue significantly depends on the system reliability, and such a situation can be observed particularly in designing structural reliability and high reliability computer systems.

(6) In both cases, the consumer selects his action which maximizes his expected profit among three options A_0, A_1 and A_n as follows:
   - option A_0: purchasing neither of the two systems;
   - option A_1: purchasing a single unit system;
   - option A_n: purchasing an n-unit system.

(7) Let $\Pi_i (i = 0, 1, n)$ and $Q_i (i = 0, 1, n)$, respectively, express the consumer's expected profit and the manufacturer's profit when the consumer selects option $A_i (i = 0, 1, n)$.

2.2. Additional assumptions and notations in Case 1

This subsection makes some additional assumptions along with additional notations indigenous to Case 1:

(8) The failure times of system units are identically and independently distributed with an exponential distribution, where the mean life of the unit is denoted by μ, and μ is a design variable in Case 1.

(9) The manufacturing cost of a system unit is expressed by $a(\mu)$, which is continuous and non-decreasing in μ.

(10) The consumer generates his revenue r per unit of usage time from a purchased system.

(11) Let $S_n = \sum_{k=1}^{n} \frac{1}{k}$ for simplicity.

In the above, we assumed an exponential distribution as an underlying unit life distribution. This is because the exponential distribution is anticipated to provide very simple results of the analysis, which will make the underpinnings of this paper more understandable, and because it is assumed for the purpose of expressing the unit life for electric and/or electronic systems. In addition, the exponential distribution is well known to be non-informative due to its memoryless property. If we can assume other life distributions which are more informative than the exponential distribution, it indicates that we can utilize such information efficiently.

2.3. Additional assumptions and notations in Case 2

The additional assumptions and notations to Case 2 against Case 1 are as follows:

(12) The reliability of a system unit is expressed by R, which is a design variable in Case 2.

(13) The manufacturing cost of a system unit is given by $a(R)$, which is continuous and non-decreasing in R.

(14) With a view to expressing an environment where the consumer's revenue remarkably depends on the system reliability, we assume the consumer's revenue is given by

$$\pi(R_s) = -r\log_{10}(1 - R_s), \quad r > 0. \tag{1}$$

It should be noted in Eq. (1) that the consumer's revenue provides $\pi(R_s) = r, 2r, 3r, \ldots$ for $R_s = 0.9, 0.99, 0.999, \ldots$

3. Analysis in Case 1

This section discusses Case 1 where the consumer's revenue is proportional to the system life. Derived is the consumer's optimal reaction which maximizes his expected profit, and then the optimal strategy of the manufacturer maximizing his profit is discussed on the basis of the consumer's optimal reaction. Two theorems are also presented regarding the manufacturer's optimal strategy.

3.1. Consumer's optimal reaction

If the consumer purchases neither a single unit system nor an n-unit system, the expected profit of the consumer becomes

$$\Pi_0 = 0. \tag{2}$$

When the consumer purchases a single unit system, the consumer's expected profit is given by

$$\Pi_1 = r\mu - P_1 \tag{3}$$

since the mean life of a single unit system is μ.

It should be noted that the mean life of an n-unit system is given by $S_n\mu$ from assumption (8) that the unit failure time follows an exponential distribution with mean μ. Hence, the consumer's expected profit by purchasing an n-unit system is expressed by

$$\Pi_n = rS_n\mu - P_n. \tag{4}$$

When $\Pi_i > \Pi_j$, the consumer prefers option A_i to option A_j where $i \neq j$ and $i, j = 0, 1, n$. This indicates that the consumer's optimal reaction becomes:

(1) in the case of $(P_n, \mu) \in \Omega_i \ (i = 0, 1, n)$, the optimal reaction is to select option A_i, and

(2) in case (P_n, μ) is located on the boundary line between Ω_i and $\Omega_j \ (i, j = 0, 1, n; i \neq j)$, option A_i is indifferent to option A_j, where $\Omega_i (i = 0, 1, n)$ are defined by the following Eqs. (5)–(7):