



The Riesz–Kantorovich formula and general equilibrium theory

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Abstract

Let L be an ordered topological vector space with topological dual L' and order dual L^\sim . Also, let f and g be two order-bounded linear functionals on L for which the supremum $f \vee g$ exists in L . We say that $f \vee g$ satisfies the Riesz–Kantorovich formula if for any $0 \leq \omega \in L$ we have

$$f \vee g(\omega) = \sup_{0 \leq x \leq \omega} [f(x) + g(\omega - x)].$$

This is always the case when L is a vector lattice and more generally when L has the Riesz Decomposition Property and its cone is generating. The formula has appeared as the crucial step in many recent proofs of the existence of equilibrium in economies with infinite dimensional commodity spaces. It has also been interpreted by the authors in terms of the revenue function of a discriminatory price auction for commodity bundles and has been used to extend the existence of equilibrium results in models beyond the vector lattice settings. This paper addresses the following open mathematical question:

- *Is there an example of a pair of order-bounded linear functionals f and g for which the supremum $f \vee g$ exists but does not satisfy the Riesz–Kantorovich formula?*

We show that if f and g are continuous, then $f \vee g$ must satisfy the Riesz–Kantorovich formula when L has an order unit and has weakly compact order intervals. If in addition L

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is locally convex, $f \vee g$ exists in L^\sim for any pair of continuous linear functionals f and g if and only if L has the Riesz Decomposition Property. In particular, if L^\sim separates points in L and order intervals are $\sigma(L, L^\sim)$ -compact, then the order dual L^\sim is a vector lattice if and only if L has the Riesz Decomposition Property — that is, if and only if commodity bundles are perfectly divisible. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

It has for sometime been well-understood that one cannot hope to prove the existence of general equilibrium — or establish the validity of the welfare theorems — under the standard finite dimensional assumptions when the commodity space is infinite dimensional and consumption sets lack interior points. In this literature, the commodity space is most often a Riesz space (vector lattice) and primitive data of the economy are supposed to satisfy various assumptions known as “properness conditions” (see Aliprantis et al., 1990, Aliprantis et al., 2000).

A distinctive feature of this literature is the non-trivial use of the lattice structure of the commodity space. Indeed, Aliprantis and Burkinshaw (1991) show that when the commodity space is a vector lattice, the lattice structure of the dual space is basically equivalent to the validity of the welfare theorems.¹ Furthermore, the various proofs in this literature can be delineated by means of the Riesz Decomposition Property of the commodity space. For example, Mas-Colell (1986) and Aliprantis et al. (1987) use the Decomposition Property to facilitate a separating hyperplane argument, while Yannelis and Zame (1986) use the property to show the continuity and extendibility of the equilibrium prices of truncated economies. This is in sharp contrast to the case where consumption sets are assumed to have interior points and where the existence of a continuous quasi-equilibrium price can be proven with little reference to the lattice structure of the commodity space (see for example, Bewley, 1972; Florenzano, 1983).

In the more recent approach of Mas-Colell and Richard (1991) and Richard (1989) (see also Deghdak and Florenzano, 1999; Podczeck, 1996; Tourky, 1998, 1999) the Decomposition Property is used in an indirect manner. Here, the authors consider economies in the more general setting of a Riesz commodity space that need not be locally solid. In this setting a supporting hyperplane argument in the space of allocations furnishes a list of prices and the crucial part of the proof is showing that the supremum of these prices is indeed the required supporting (equilibrium) price. In this second group of papers, the Decomposition Property is used through two of its consequences. First, the fact that the order dual of the

¹ Of course, here we are talking about those welfare theorems that are traditionally proven using a separating hyperplane argument, i.e., the second welfare theorem and the equivalence of Edgeworth and Walrasian equilibria.

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