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General Equilibrium When Economic Growth Exceeds Discounting[1]

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After dropping the standard general-equilibrium assumption that preference orders discount future consumption faster than the economy grows and dropping continuity and weakening utility representation, we establish commodity prices and consumptions that approach approximate equilibrium to within any practical tolerance. The Weizäcker-overtaking criterion defines the best-known non-standard-discounting orders we admit over discrete-time, deterministic consumption paths and over continuous-time, stochastic consumption processes. We also perturb preferences to qualify all approximate equilibrium as full equilibrium, thus showing some well-known non-existence examples are singular, and so are inadequate defence of standard assumptions. *Journal of Economic Literature* Classification Numbers: C60, C62.

1. INTRODUCTION

We admit new preference orders to general-equilibrium theory by finding a type of limit of approximate equilibrium without assuming orders discount future consumption faster than the economy grows. For a discrete-time deterministic example, if the economy actually grows but consumption units are normalized to keep endowments at 1 each period, then we admit orders that negatively discount, or *up*-count, normalized units in the sense that 1 normalized unit of future consumption (the future endowment) is preferred to 1 normalized unit of current consumption (the current endowment). Roughly, any discounting that would have been caused by impatience for consumption in actual units is *offset* by the growing endowment in actual units. The Weizäcker-overtaking criterion defines the best-known up-counting orders over any infinite-horizon commodity space. For

[1] This paper extends an earlier, unpublished analysis of limit equilibrium for non-discounting preferences (Burke [4]) from discrete-time, deterministic commodities to general infinite-dimensional commodity spaces, while offering stronger conclusions. Thanks to an anonymous referee for careful proofreading and provocative questions.

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example, discrete-time, deterministic consumption path $(x_t) \geqslant 0$ in l_∞ overtakes path $(\hat{x}_t)$ when partial sums $\sum_{t=1}^{T} 2^t(x_t - \hat{x}_t) > 0$ for large T. Likewise, continuous-time, stochastic consumption process $(X(t)) \geqslant 0$ in a suitable L_∞-space (Subsection 2.2) overtakes process $(\hat{X}(t))$ when partial (Lebesgue) integrals $\int_0^T 2^t(EX(t) - E\hat{X}(t))\,dt > 0$ for large T, where $EX(t)$ is the expected value from stochastic consumption $X(t)$ at time $t \in [0, \infty)$.

The reason general-equilibrium theorist's assume discounting is faster than growth are well-known examples showing the possibility of non-existence without discounting, and the failure of standard existence theorems and proofs without discounting. For example, consider a 2-person, discrete-time, deterministic, pure-exchange economy with 1 unit endowment for each consumer in each period. Despite satisfying all standard assumptions except discounting, the economy has no (exact) equilibrium when one consumer has discounted utility $u(x) = \sum_1^\infty 2^{-t}x_t$ over l_∞^+, and the other has an up-counting Banach limit $v(x) = \operatorname{Lim} x_t$ [2]. In fact, for each period, transferring consumption from the up-counting consumer to the discounting consumer Pareto-improves utility; hence, the only Pareto optimum and only candidate for equilibrium allocates the up-counting consumer zero each period, which violates standard individual rationality. However, the zero-consumption allocation is a type of limit of approximate equilibrium under the price system $p(x) = \sum_1^\infty 2^{-t}x_t + \operatorname{Lim} x_t$, where the Banach limit $(\operatorname{Lim} x_t)$ is a norm-continuous positive linear function over l_∞ that defines a price bubble on certain infinite-lasting consumption and endowment paths. Precisely, under p, each consumer's unit endowment has value 2 and the supremum of each utility over the budget set is also 2. And supremum utility levels are approached by the approximate-equilibrium allocation of zero to the up-counting consumer for the first 100 years followed by the total endowment thereafter (generating utility $v(x) = \operatorname{Lim} x_t = 2$), with the total endowment followed by zero for the discounting consumer (generating utility $u(x) = \sum_1^\infty 2^{-t}x_t = \sum_1^{100} 2^{-t}2 = 2 - 2^{-99}$).

More generally, for a mixture of up-counting and discounting preference orders among a finite number of consumers of continuous-time stochastic consumption processes, which include discrete-time and deterministic consumption as a special case, we will find a price system p and an allocation (x_i) of consumptions that approach approximate equilibrium to within any practical tolerance. For example, partition the first hundred years into one-second intervals and partition states of nature into one hundred events E. Hence, for each positive tolerance ε, under price system p there exists an ε-approximate equilibrium allocation (x_i^ε) for which, when restricted to time in any given 1-second interval and states in any given event E, each consumer's average consumption in x_i^ε is within ε of average consumption in x_i. Thus we offer limit-equilibrium price system p and