



On the structural behavior and the Saint Venant solution in the exact beam theory Application to laminated composite beams

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Abstract

Based on the exact beam theory, the paper focuses on the effective calculation of the characteristic operators involved in beam's structural behavior and Saint Venant solution. It is shown that these operators can be derived from the solutions of six characteristic beam elasticity problems. These solutions are determined by minimizing potential energy functionals which allow a finite element computation technique where only the cross-section has to be discretized. This computation technique is applied to typical symmetric laminated composite beams, and to antisymmetric ones. Structural beam rigidities and couplings, warpings, and 3D stresses (including interlaminar stresses and free edge-effects) are provided and compared to available results. The proposed computation technique appears as an accurate and easy tool allowing an effective use of the exact beam theory.

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1. Introduction

The exact beam theory is established by Ladevèze and Simmonds [8]. It is valid for straight prismatic beams, piecewise constant cross-sections, heterogeneous and anisotropic elastic materials, arbitrary loadings, and any shape ratios. Since it is independent of any kinematic or static assumption, this theory is quite different from the classical theories of Euler–Bernoulli, Timoshenko [18], and their extensions. In the exact beam theory, the solution of a 3D elasticity beam problem is viewed as the sum of a long wavelength part and a localized short wavelength part. Such partition was also

used in the works of Toupin [17], Ladeveze [7], Horgan [4] and Horgan and Simmonds [5] on the Saint Venant's principle. The long wavelength part, called the Saint Venant solution, has a fundamental role in the exact beam theory. Indeed, it constitutes the interior part of the solution and it is also needed for the evaluation of local effects. The expression of the Saint Venant solution, which is given in Section 2, involves the classical cross-sectional stress resultants, cross-sectional displacements and rotations, and characteristic operators depending on the cross-section geometry and the materials. The cross-sectional stress resultants, displacements, and rotations are solution of a 1D elastic beam theory which includes a beam's compliance operator. This operator describes the beam's behavior at the structural level. The present paper focuses on the effective calculation of the different operators involved in the 3D Saint Venant solution and the 1D structural elastic behavior. The approach developed here is different from

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the one based on a 3D problem and proposed by Ladevèze et al. [9] and Sanchez et al. [16]. In the current paper the operators are directly derived from the solutions of a set of 2D problems, which are presented in Section 3.

Laminated composite beams are known to exhibit complex phenomena such that coupled deformations arising from the orthotropic nature of the laminae and from the stacking sequences. Detailed structural models are then essential in order to fully exploit such special effects in design. In addition to the structural level, detailed 3D stress analysis is of practical relevance for laminated composites and especially the interlaminar stresses which may result in delamination and failure of the laminates. Indeed, the occurrence of high localized interlaminar stresses along the free edges of composite laminates is a well-known phenomenon that has been recognized for almost 30 years now. Consequently, many refined theories have been developed in order to describe the behavior of laminated composites (e.g. [1,2,13–15]) and references therein). These theories are generally built upon simplifying assumptions which vary with the studied effect and the author. Since the pioneering analysis of Pipes and Pagano [12], numerous papers including finite element analyses were devoted to 3D state of stresses and free edge effects in composite laminates for both mechanical and thermal loadings (e.g. [3,10,11,19,20] and references therein). Structural behavior, warping, and 3D stresses in symmetric and in antisymmetric typical Graphite/Epoxy laminated beams are analyzed in Section 4. This analysis is based on the exact beam theory.

2. Background of the exact beam theory

For a beam of e_x axis, the analysis of a 3D beam elasticity problem with x -constant data is the starting point of the exact beam theory. The beam is occupying a prismatic domain Ω of x -constant cross-section S and a length L (Fig. 1). S_{lat} is the lateral surface. S_0 and S_L are the extremity sections. e_y and e_z are the inertia unit vectors of the cross-section. A point M in Ω is marked $M = xe_x + X$, where X belongs to S (vectors are noted in boldface characters). The materials constituting the beam are linear elastic and the elastic tensor field K is x -constant. The beam is subjected to a body force density

f^d on Ω , and surface force densities F^d , H_0 and H_L on S_{lat} , S_0 and S_L , respectively. The densities f^d and F^d are x -constant. The equations of the linearized equilibrium problem are:

$$\text{div}(\sigma) + f^d = \mathbf{0} \quad \text{on } \Omega \tag{1}$$

$$\varepsilon(\xi) = \frac{1}{2}(\nabla' \xi + \nabla \xi) \quad \text{on } \Omega \tag{2}$$

$$\sigma = K(\varepsilon) \quad \text{on } \Omega \tag{3}$$

$$\sigma(\mathbf{n}) = F^d \quad \text{on } S_{lat} \tag{4}$$

$$\sigma(-e_x) = H_0 \quad \text{on } S_0 \tag{5}$$

$$\sigma(e_x) = H_L \quad \text{on } S_L \tag{6}$$

where ξ is the displacement vector, σ is the stress tensor, and \mathbf{n} is the unit vector that is normal and external to S_{lat} . The solution of this problem is denoted by $\langle s \rangle = \langle \sigma, \xi \rangle$. Note that ξ is unique only within an arbitrary rigid body displacement.

The Saint Venant solution, denoted by $\langle s^{sv} \rangle = \langle \sigma^{sv}, \xi^{sv} \rangle$, is an x -polynomial solution of the Eqs. (1)–(4). It satisfies the boundary conditions (5) and (6) only in terms of resultant and moment of the normal stresses acting on the extremity sections [6]. The expression of the Saint Venant solution, as given by Ladevèze and Simmonds [8], is:

$$\xi^{sv} = \mathbf{u}(x) + \omega(x) \wedge X + \mathcal{A}(X)T(x) + \mathcal{B}(X)M(x) + W^d(X) \tag{7}$$

$$\sigma^{sv}(e_x) = \mathcal{A}^0(X)T(x) + \mathcal{B}^0(X)M(x) + C^d(X) \tag{8}$$

T and M are the classical cross-sectional stress resultants,

$$T(x) = \int_S \sigma(e_x) dS, \quad M(x) = \int_S X \wedge \sigma(e_x) dS, \tag{9}$$

which verify the beam equilibrium equations:

$$\begin{cases} T_{,x} + p^d = \mathbf{0} \\ M_{,x} + e_x \wedge T + \mu^d = \mathbf{0} \end{cases} \tag{10}$$

where $(\cdot)_{,x}$ is the derivative with respect to x . $[p^d, \mu^d]$ are the lineic force densities associated with $[f^d, F^d]$:

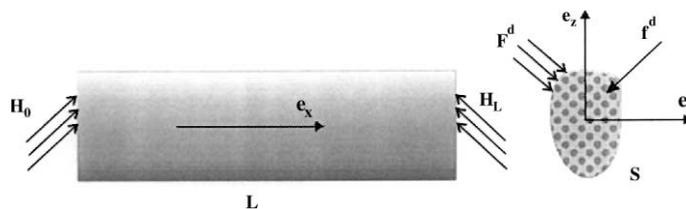


Fig. 1. Geometry and loading of the beam.

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