Mechanical Systems and Signal Processing (2003) **17**(1), 243–250
doi:10.1006/mssp.2002.1557, available online at http://www.idealibrary.com on IDEAL®

INSTANTANEOUS INDICATORS OF STRUCTURAL BEHAVIOUR BASED ON THE CONTINUOUS CAUCHY WAVELET ANALYSIS

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(Received 1 October 2002)

In this contribution, four instantaneous indicators are proposed to characterize the non-linear behaviour of mechanical structures from their transient responses. These indicators are developed from the Cauchy wavelet analysis of the accelerometric responses of the structure. They are based on the notion of ridges and skeletons introduced for the wavelet processing of signals that are assumed to be defined as a sum of asymptotic amplitude and phase-modulated terms. The proposed indicators are then applied to the data collected at the University of Liège. Some preliminary results are given for the characterization of the non-linear behaviour of the beam.

1. INTRODUCTION

Since its first definition in the early 1980s by French researchers, especially Grossmann and Morlet [1], the growth of wavelet research in mathematics has been exponential with significant contributions from numerous authors and its application in signal and image processing has been successful. Although some authors (see reference [2]) highlighted the potential benefits of using wavelets for the analysis of engineering data, wavelet analysis is still far from being systematically used in mechanical engineering, certainly due to the ignorance of the properties of the wavelet transform. Our aim is to promote the use of wavelet analysis for processing the time variation of the spectral content of transient responses of mechanical structures. The use of a time–frequency representation of measured signals is more adapted in the sense that it highlights the maximum amount of information within the data and it makes the parameter identification procedure much easier. The continuous Cauchy wavelet transform (CCWT) is defined and instantaneous indicators, based on the properties of the CCWT applied to asymptotic signals, are proposed to characterize the 'modal' behaviour of the structure under testing. Accelerometric responses of buildings under non-destructive shocks have been processed with the CCWT in reference [3] for the assessment of the modal characteristics of buildings. Here, the procedure is applied to the accelerometric responses of a non-linear beam to an impact force obtained in the LTAS-VIS of the University of Liège.

2. THE CONTINOUS CAUCHY WAVELET TRANSFORM

Let us introduce the complex-valued Cauchy wavelet $\psi_n \in L^1(\mathbb{R}) \bigcap L^2(\mathbb{R})$ of n order for $n \geqslant 1$

$$\psi_n(t) = \left(\frac{i}{t+i}\right)^{n+1} = \left[\frac{1}{\sqrt{t^2+1}}\right]^{n+1} e^{i\,(n+1)\mathrm{Arctg}(t)}. \tag{1}$$

The Fourier transform of $\psi_n(t)$ is: $\hat{\psi}_n(\omega) = (2\pi\omega^n e^{-\omega}/n!)\, H(\omega)$ where $H(\omega)$ is the Heaviside function. Cauchy wavelets are admissible and progressive: $\hat{\psi}_n(\omega) = 0 : \forall\omega \leqslant 0$. The CCWT of a finite energy signal $u(t)$ is defined as the wavelet transform of $u(t)$ when the analysing wavelet is the Cauchy wavelet of order n :

$$T_{\psi_n}[u](b,a) = \frac{1}{a}\int_{-\infty}^{+\infty} u(t)\,\bar{\psi}_n\left(\frac{t-b}{a}\right) dt. \tag{2}$$

It is note worthy that the CCWT has the following property:

$$T_{\psi_n}[\ddot{u}_k(t)](b,a) = -\frac{(n+2)(n+1)}{a^2} T_{\psi_{n+2}}[u_k(t)](b,a). \tag{3}$$

3. THE CCWT OF A SUM OF ASYMPTOTIC SIGNALS

This section deals with signals $u(t)$ expressed as a sum of M asymptotic real, frequency and amplitude modulated signals $u_k(t)$ of finite energy

$$u(t) = \sum_{k=1}^{M} u_k(t) = \sum_{k=1}^{M} A_k(t)\cos(\alpha_k(t)) \tag{4}$$

with amplitude term $A_k(t)$ positive and phase term $\alpha_k(t) \in [0, 2\pi)\ \forall t \in \mathbb{R}$. The variation of each instantaneous frequency $\dot{\alpha}_k(t)$ is assumed not to interfere from the others. Carmona *et al.* give the definition of a class of signals called *asymptotic* and present (see reference [3]) theoretical and numerical results for the time–frequency analysis of such signals. An asymptotic signal $u_k(t)$ is oscillatory enough so that its associated analytic signal $U_k(t)$ can be approximated in the form: $U_k(t) \simeq A_k(t)\, e^{i\alpha_k(t)}$ which essentially means that the oscillations coming from $\alpha_k(t)$ are much faster than the variations coming from $A_k(t)$. The main feature of the CCWT of $u_k(t)$ is that it is concentrated along one curve in the time–frequency domain called 'ridge' and labelled $a_{rk}(b)$ and that the restriction of the CCWT to the ridge called the 'skeleton' is very close to the signal $u_k(t)$ itself. Carmona *et al.* show in [4] that

$$T_{\psi_n}[u_k(t)](b,a) \simeq \tfrac{1}{2}\overline{\hat{\psi}_n}(a\dot{\alpha}_k(b))\, U_k(t). \tag{5}$$

Due to the linearity of the CCWT, we have then

$$T_{\psi_n}[u(t)](b,a) \simeq \tfrac{1}{2}\sum_{k=1}^{M} \overline{\hat{\psi}_n}(a\dot{\alpha}_k(b))\, A_k(b)\, e^{i\alpha_k(b)}. \tag{6}$$

There are several ways to define and then to extract a ridge. Here, for each value of b, we look for the value of a such as $|T_{\psi_n}[u_k(t)](b, a_{rk}(b))| = \max_a |T_{\psi_n}[u_k(t)](b,a)|$. The ridge and the associated skeleton are then given by

$$a_{rk}(b) = \frac{n}{\dot{\alpha}_k(b)} \tag{7}$$

$$T_{\psi_n}[u(t)](b, a_{rk}(b)) \simeq \pi\frac{n^n e^{-n}}{n!} A_k(b)\, e^{i\alpha_k(b)}. \tag{8}$$