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Solving dynamic general equilibrium models using a second-order approximation to the policy function

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Abstract

This paper derives a second-order approximation to the solution of a general class of discrete-time rational expectations models. The main theoretical contribution is to show that for any model belonging to that class, the coefficients on the terms linear and quadratic in the state vector in a second-order expansion of the decision rule are independent of the volatility of the exogenous shocks. In addition, the paper presents a set of MATLAB programs that implement the proposed second-order approximation method and applies it to a number of model economies.

JEL classification: E0; C63

Keywords: Solving dynamic general equilibrium models; Second-order approximation; Matlab code

1. Introduction

Since the seminal papers of Kydland and Prescott (1982) and King et al. (1988), it has become commonplace in macroeconomics to approximate the solution to non-linear, dynamic, stochastic, general equilibrium models using linear methods. Linear approximation methods are useful to characterize certain aspects of the dynamic properties of complicated models. In particular, if the support of the shocks driving aggregate fluctuations is small and an interior stationary solution exists, first-order approximations provide adequate answers to questions such as local existence and determinacy of equilibrium and the size of the second moments of endogenous variables.

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However, first-order approximation techniques are not well suited to handle questions such as welfare comparisons across alternative stochastic or policy environments. For example, Kim and Kim (in press) show that in a simple two-agent economy, a welfare comparison based on an evaluation of the utility function using a linear approximation to the policy function may yield the spurious result that welfare is higher under autarky than under full risk sharing. The problem here is that some second- and higher-order terms of the equilibrium welfare function are omitted while others are included. Consequently, the resulting criterion is inaccurate to order two or higher. The same problem arises under the common practice in macroeconomics of evaluating a second-order approximation to the objective function using a first-order approximation to the decision rules. For in this case, too, some second-order terms of the equilibrium welfare function are ignored while others are not.[1] In general, a correct second-order approximation of the equilibrium welfare function requires a second-order approximation to the policy function.

In this paper, we derive a second-order approximation to the policy function of a general class of dynamic, discrete-time, rational expectations models. A strength of our approach is not to follow a value function formulation. This allows us to tackle easily a wide variety of model economies that do not lend themselves naturally to the value function specification. To obtain an accurate second-order approximation, we use a perturbation method that incorporates a scale parameter for the standard deviations of the exogenous shocks as an argument of the policy function. In approximating the policy function, we take a second-order Taylor expansion with respect to the state variables as well as this scale parameter. This technique was formally introduced by Fleming (1971) and has been applied extensively to economic models by Judd and co-authors (see Judd, 1998, and the references cited therein).

The main theoretical contributions of the paper are: First, it shows analytically that in general the first derivative of the policy function with respect to the parameter scaling the variance/covariance matrix of the shocks is zero at the steady state regardless of whether the model displays the certainty-equivalence property or not.[2] Second, it proves that in general the cross derivative of the policy function with respect to the state vector and with respect to the parameter scaling the variance/covariance matrix of the shocks evaluated at the steady state is zero. This result implies that for any model belonging to the general class considered in this paper, the coefficients on the terms linear and quadratic in the state vector in a second-order expansion of the decision rule are independent of the volatility of the exogenous shocks. In other words, these coefficients must be the same in the stochastic and the deterministic versions of the model. Thus, up to second order, the presence of uncertainty affects only the constant term of the decision rules.

[1] See Woodford (2002) for a discussion of conditions under which it is correct up to second order to approximate the level of welfare using first-order approximations to the policy function.

[2] Judd (1998, pp. 477–480) obtains this result in the context of a simple one-sector, stochastic, discrete-time growth model. Thus, our theoretical finding can be viewed as a generalization of Judd's result to a wide class of rational expectations models.