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General equilibrium comparative statics: discrete shocks in production economies

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Abstract

Nachbar [Econometrica 79 (5) (2002) 2065] established minimal conditions under which, following an infinitesimal shock to endowments in an exchange economy, changes in equilibrium prices are negatively related to changes in aggregate consumption. The present paper extends Nachbar (2002) to cover discrete shocks to technologies, ownership shares, and endowments in production economies. As in Nachbar (2002), the analyst's choice of price normalization plays a key role. The required normalization is nonstandard but has a sensible interpretation.

Keywords: General equilibrium; Comparative statics; Weak Axiom

1. Introduction

Consider a shock to technologies, ownership shares, or endowments in a production economy. A plausible conjecture is that

$$(p^{\diamondsuit} - p^{*}) \cdot (\bar{x}^{\diamondsuit} - \bar{x}^{*}) \leq 0,$$

where $p^{\diamondsuit} - p^{*}$ is the change in equilibrium prices and $\bar{x}^{\diamondsuit} - \bar{x}^{*}$ is the change in aggregate equilibrium consumption. Informally, changes in equilibrium prices are negatively related to changes in aggregate equilibrium consumption. It has been known at least since Hicks (1939) that this conjecture is false, in general, even in single consumer exchange economies. This paper provides minimal conditions under which the conjecture is, in fact, true.

This paper is an extension of Nachbar (2002), which provided analogous results for infinitesimal shocks to endowments in exchange economies. Since the discrete results are easiest to interpret in relation to the infinitesimal results of Nachbar (2002), I begin by reviewing the earlier paper.

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The analysis in Nachbar (2002) requires that, regardless of how prices are actually normalized in the economy, prices be renormalized to satisfy $p \cdot \bar{\mu} = 1$, where $\bar{\mu}$ is the vector giving the economy's aggregate *marginal propensity to consume* (MPC) each of the L goods. In a single consumer economy, the MPC is simply the derivative with respect to nominal wealth of that consumer's demand. In multiconsumer economies, $\bar{\mu}$ is a weighted sum of individual MPCs.

The aggregate MPC price normalization is nonstandard. It has the unattractive property that it depends both on the reference equilibrium and, in multiconsumer economies, on the distribution of the endowment shocks. And estimating the aggregate MPC may be difficult. In defense of the aggregate MPC price normalization, Nachbar (2002) makes two observations.

First, the aggregate MPC price normalization is necessary in the following sense. In single consumer economies, the MPC price normalization is the unique linear price normalization (up to scalar multiplication) for which $(p^{\diamondsuit} - p^*) \cdot (\bar{x}^{\diamondsuit} - \bar{x}^*) \leq 0$ holds for all endowment shocks. A similar but weaker uniqueness claim holds in multiconsumer economies.

Second, if $\bar{\mu} > 0$ (all goods are weakly normal in the aggregate) then the $\bar{\mu}$ normalization yields a sensible interpretation of the vector of normalized price changes.[1] A decrease in the $\bar{\mu}$-normalized price of good 1 implies that the price of good 1 has fallen relative to the value of the composite commodity $(\bar{\mu}_2, \dots, \bar{\mu}_L) > 0$, which is the economy's marginal consumption of all other goods. Since the comparative statics result in Nachbar (2002)considers a marginal change in endowment, interpreting relative price changes in terms of marginal consumption is, I think, natural. If, on the other hand, $\bar{\mu} \not> 0$ (some goods are inferior in the aggregate) then the vector of normalized price changes may be difficult to interpret; see Nachbar (2002).

I have glossed over the fact that, in multiconsumer economies, it is unclear what weighting scheme one should use for computing $\bar{\mu}$. Nachbar (2002) actually considers two different weighting schemes, yielding two different aggregate MPCs, $\bar{\mu}_\gamma$ and $\bar{\mu}_\delta$. For $\bar{\mu}_\gamma$, the weight on i's individual MPC is based on the value, at the original equilibrium prices, of the change in i's equilibrium consumption. For $\bar{\mu}_\delta$, the weight on i's individual MPC is based on the value, at the original equilibrium prices, of the change in i's endowment. The $\bar{\mu}_\gamma$ and $\bar{\mu}_\delta$ normalizations yield price change vectors corresponding to somewhat different interpretations of what one means by a relative price change. I find both interpretations reasonable.

Relative to Nachbar (2002), the contribution of the present paper is three-fold.

1. This paper provides comparative statics results for discrete, as opposed to infinitesimal, shocks to endowments in exchange economies. This extension is straightforward but still worth recording.[2] Corresponding to the aggregate MPC vectors $\bar{\mu}_\gamma$ and $\bar{\mu}_\delta$ in Nachbar (2002), this paper uses the *incremental propensity to consume* (IPC) vectors $\bar{\mu}_x^\Delta$ and $\bar{\mu}_\omega^\Delta$. The interpretation of an IPC normalized price change is analogous to the interpretation of an MPC normalized price change.

[1] $\mathbb{R}^L_+ = \{x \in \mathbb{R}^L : x_\ell \geq 0,\ \forall \ell\}$, $\mathbb{R}^L_{+\backslash 0} = \{x \in \mathbb{R}^L_+ : x \neq 0\}$, and $\mathbb{R}^L_{++} = \{x \in \mathbb{R}^N : x_\ell > 0,\ \forall \ell\}$. Consider any $x \in \mathbb{R}^L$. Then $x \geq 0$ means $x \in \mathbb{R}^L_+$, $x > 0$ means $x \in \mathbb{R}^L_{+\backslash 0}$, and $x \gg 0$ means $x \in \mathbb{R}^L_{++}$.

[2] A discrete extension of the basic inequalities in Nachbar (2002) has been independently reported in Quah (2003).