Global Instability in Experimental General Equilibrium: The Scarf Example

Christopher M. Anderson* Charles R. Plott†
Ken-Ichi Shimomura‡ Sander Granat§

April 10, 2003

0.1 Relative Price Adjustment

While microeconomists typically use the absolute price adjustment specification considered by Scarf, macroeconomists often assume that price adjustments are proportional to the price level. In this economy, this assumption means prices adjust according to the following simultaneous differential equations:

$$\frac{\dot{p}_x}{p_x} = \lambda_x E^x(p_x, p_y) \quad (1)$$

$$\frac{\dot{p}_y}{p_y} = \lambda_y E^y(p_x, p_y) \quad (2)$$

where $\lambda_x > 0$ and $\lambda_y > 0$.

*Assistant Professor of Environmental and Natural Resource Economics, University of Rhode Island, cma@uri.edu.

†Edward S. Harkness Professor of Economics and Political Science, California Institute of Technology. cplott@hss.caltech.edu

‡Associate Professor of Economics, OSIPP, Osaka University. ken-ichi@osipp.osaka-u.ac.jp.

§Undergraduate, California Institute of Technology.

0.1.1 Instability

As with the absolute specification, the system of Equations (1) and (2) is globally unstable if $\alpha + \beta = 1,$ and $\gamma = 0$.

Following the same method to integrate the system as in the absolute adjustment specification, the system can be written

$$\begin{aligned} \frac{\dot{p_x}}{\lambda_x \omega_x p_x} &= \frac{p_x(1-\alpha)\omega_x + \alpha\omega_z}{p_x\omega_x + \omega_z} + \frac{p_x\alpha\omega_x + p_y(1-\alpha)\omega_y}{p_x\omega_x + p_y\omega_y} - 1 \\ &= \frac{p_x\omega_x(1-2\alpha)(p_y\omega_y - \omega_z)}{(p_x\omega_x + \omega_z)(p_x\omega_x + p_y\omega_y)} \end{aligned} \tag{3}$$

$$\begin{aligned} \frac{\dot{p_y}}{\lambda_y \omega_y p_x} &= \frac{p_y\alpha\omega_y + (1-\alpha)\omega_z}{p_y\omega_y + \omega_z} + \frac{p_x\alpha\omega_x + p_y(1-\alpha)\omega_y}{p_x\omega_x + p_y\omega_y} - 1 \\ &= \frac{p_y\omega_y(2\alpha-1)(p_x\omega_x - \omega_z)}{(p_y\omega_y + \omega_z)(p_x\omega_x + p_y\omega_y)}. \end{aligned} \tag{4}$$

Both Equation (3) and Equation (4) can be manipulated to find an expression which is equal to $(1-2\alpha)(p_y\omega_y - \omega_z)(p_x\omega_x - \omega_z)/(p_x\omega_x + p_y\omega_y)$. Subtracting these two expressions gives

$$\begin{aligned} \frac{\dot{p_x}}{\lambda_x (p_x)^2 (\omega_x)^2}(p_x\omega_x + \omega_z)(p_x\omega_x - \omega_z) + \frac{\dot{p_y}}{\lambda_y (p_y)^2 (\omega_y)^2}(p_y\omega_y + \omega_z)(p_y\omega_y - \omega_z) &= 0 \\ \frac{\dot{p_x}}{\lambda_x}\left(1 - \left(\frac{\omega_z}{\omega_x}\right)^2 \frac{1}{(p_x)^2}\right) + \frac{\dot{p_y}}{\lambda_y}\left(1 - \left(\frac{\omega_z}{\omega_y}\right)^2 \frac{1}{(p_y)^2}\right) &= 0 \end{aligned}$$

This equation can be directly integrated with respect to t to give a function which describes the paths along which prices adjust, the solution to the system of Equations (3) and (4). In particular,

$$\frac{1}{\lambda_x}\left(p_x + \left(\frac{\omega_z}{\omega_x}\right)^2 \frac{1}{p_x}\right) + \frac{1}{\lambda_y}\left(p_y + \left(\frac{\omega_z}{\omega_y}\right)^2 \frac{1}{p_y}\right) = C$$

where C is the constant value of the level surface on which (p_x, p_y) lies.