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General equilibrium analysis in ordered topological vector spaces

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Abstract

The second welfare theorem and the core-equivalence theorem have been proved to be fundamental tools for obtaining equilibrium existence theorems, especially in an infinite dimensional setting. For well-behaved exchange economies that we call proper economies, this paper gives (minimal) conditions for supporting with prices Pareto optimal allocations and decentralizing Edgeworth equilibrium allocations as non-trivial quasi-equilibria. As we assume neither transitivity nor monotonicity on the preferences of consumers, most of the existing equilibrium existence results are a consequence of our results. A natural application is in Finance, where our conditions lead to new equilibrium existence results, and also explain why some financial economies fail to have equilibria.

JEL classification: C61; C65; D41; D51; G11; G12

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1. Introduction

The welfare theorems and the core-equivalence theorem state important optimal properties of general equilibrium allocations. Loosely speaking, given the technological and resource limitations, individual needs and tastes that define an economy, every equilibrium allocation is as well a Pareto optimal allocation, a core allocation and an Edgeworth

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equilibrium. Converse results give conditions under which every Pareto optimal allocation can be supported by a non-zero price linear functional such that each consumer minimizes her expenditure and each firm maximizes its profit while every Edgeworth equilibrium is a quasi-equilibrium allocation when associated with an appropriate non-zero price linear functional. These converse results have been proved to be the fundamental tools for obtaining equilibrium existence theorems. Directly or indirectly, their validity in an infinite dimensional setting generally relies either, as in Debreu (1954), on an interiority property of the positive cone of the commodity space (assumed to be an ordered topological vector space) or on the so-called properness assumptions made on the characteristics of the economy. In the latter case, the lattice structure of the commodity space plays an essential role in establishing decentralization results.

After the seminal paper of Mas-Colell (1986a), the lattice requirements have been weakened and it is now well-known that for an equilibrium existence theorem, the topology of the vector lattice commodity space need not be locally solid. However, in Deghdak and Florenzano (1999), Florenzano and Marakulin (2001), Mas-Colell (1986b), Mas-Colell and Richard (1991), Podczeck (1996), Tourky (1998), the commodity space is still assumed to be a vector lattice and the lattice properties are used in the proofs. The aim of this paper is to look at what extent the lattice structure itself of the commodity space can be given up. More precisely, its aim is to state the (minimal) properties that have to be assumed on the order structure of the commodity space for guaranteeing the validity of the decentralization theorems.

A first attempt was done in Aliprantis et al. (2001) where the characterization of Pareto optimality and Edgeworth equilibrium is given using non-linear prices for decentralizing the relevant allocations. This paper goes further and looks for decentralization with linear prices. We deal with well-behaved exchange economies that we call proper economies. Such economies, defined on a locally convex Hausdorff ordered topological vector space with topologically bounded order intervals, have consumption sets equal to the positive orthant, (semi-) continuous or concave preferences, preferred sets extendable to the entire space as convex sets with a nonempty interior, and the positive total endowment as a common properness direction. We show that necessary and/or sufficient conditions for supporting with continuous linear functionals weakly Pareto optimal allocations and decentralizing with continuous linear prices Edgeworth equilibria of any well-behaved exchange economy can be expressed in terms of the properties of the Riesz–Kantorovich formula associated with a finite list of linear functionals on the commodity space. These properties should be understood as a condition of compatibility between the topology and the order structure of the commodity space. They are implied by the assumption currently made that the commodity space is a vector lattice equipped with a locally convex topology whose positive cone is closed and the topological dual is a vector sublattice of the order dual. They are a-fortiori implied by the assumption that the commodity space is a topological vector lattice, so that we get as consequence of our main results most of earlier results for an exchange economy.

At this point, it is worth recalling that weakly Pareto optimal allocations and Edgeworth equilibria exist under mild conditions on well-behaved economies (for statements and proofs in an infinite dimensional setting, see for example Aliprantis et al. (1990), Aliprantis et al. (2001), Florenzano (1990), Mas-Colell (1986a); see also Allouch and Florenzano (2001) for a proof of the existence of Edgeworth equilibria in economies with non-compact attainable sets). Their existence does not require any lattice property of the commodity space.