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A general equilibrium analysis of strategic arbitrage

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Abstract

We analyze a general equilibrium framework with Cournot arbitrageurs and with price-taking investors who are subjected to restricted participation constraints. Restricted participation may leave some arbitrage opportunities unexploited by investors.

We show existence of Cournot–Walras equilibria with an endogenous number of arbitrageurs. The number of arbitrageurs is endogenous since they have to sink entry costs in order to arbitrage across the relevant markets. We characterize equilibria and analyze the effects on equilibrium prices and quantities of increased competition among arbitrageurs due to lower entry costs.

JEL classification: C72; D43; D5; G10

Keywords: Arbitrage; General equilibrium; Restricted participation; Cournot–Walras

1. Introduction

In this paper, we set out to describe a simple modeling framework for financial problems and to study the existence of equilibria as well as some equilibrium comparative statics.

The model was motivated by the need to model the interaction between a large number of small investors and depositors on one hand, and an endogenous number of large strategic financial players, called *arbitrageurs* or *financial intermediaries*, on the other hand. As a stylized fact, individual investors have access to restricted investment opportunities compared to the universe of investments that the large financial institutions (e.g. investment banks, mutual funds, hedge funds, etc.) can trade and invest in. This advantage allows them to profit from the inefficiencies in the market by arbitraging across the various market places, or exchanges, which explains why we refer to them as arbitrageurs. Arbitrageurs are not endowed with any capital in order to guarantee that the unique source of their profits is the exploitation of arbitrage opportunities across exchanges. An exchange (or market

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place) is formally defined as a market where a certain subset of assets is traded. There is one auctioneer per exchange.

This asymmetry of investment opportunities is captured in this framework by resorting to two assumptions. First, we restrict the participation of investors to one exchange, while the arbitrageurs can, upon sinking the required fixed investment cost, trade across all market places. This assumption is strong, but has proved useful in financial economics, and some of its applications are reviewed below where we discuss the related literature. Because investors on different exchanges cannot trade directly between themselves, they will trade indirectly via the arbitrageurs who thereby cash in the difference in the marginal willingnesses to pay in form of an assured arbitrage profit. This compensation is reminiscent of bid-ask spreads, so arbitrageurs can be viewed as intermediaries. The second assumption concerns the game form underlying the model: while investors are small and price-taking, arbitrageurs are large and play a Cournot game among themselves, taking the price impacts of their decisions into account. The equilibrium concept is therefore the Cournot–Walras equilibrium (CWE), introduced by Gabszewicz and Vial (1972) and extended and elaborated on in Hart (1979), Novshek and Sonnenschein (1978) and Roberts (1980), and surveyed by Mas-Colell (1982).

Here, we describe a general economy, with an arbitrary but finite number of exchanges, each one of which is inhabited by an arbitrary, but finite, number of price-taking investors that are restricted to trade on their own exchange only. The number of strategic and non-price-taking arbitrageurs à la Cournot, who can trade across all market places provided they have sunk the required fixed costs, is endogenous because of free (but costly) entry. This framework covers many restricted participation models encountered in the financial economics literature. The main raison d'être of this paper is therefore to set up a sufficiently general framework that encompasses and generalizes many useful models (some of which we shall review below), but that still admits equilibria and permits fairly explicit and qualitatively unambiguous comparative statics results that can be used to address explicit financial problems.

This unfortunately requires some strong assumptions, both in view of known problems with the existence of equilibria, and due to the fact that we intend to stay within a class that allows transparent comparative statics properties. The most restrictive assumption we impose is downward sloping demand, which guarantees that the Walrasian correspondence is single-valued and, importantly, continuous. We therefore do not aim at the greatest level of generality. It is well-known that the existence of Cournot–Walras equilibria is a delicate issue, for instance as pointed out by the counterexamples in Dierker and Grodal (1986) (due to the multi-valuedness of the Walrasian correspondence and the non-existence of a continuous selection) or in Roberts and Sonnenschein (1977) (due to the non-quasi-concavity of the optimization problem). We show that equilibria exist, at least if demand is downward sloping and if entry costs are low enough so that competition is intense enough. The flavor of this result is reminiscent of the conditions for existence of equilibria in Novshek and Sonnenschein (1978) and Roberts (1980), as well as in the literature that these papers generated. There the high level of competition required to show existence is guaranteed by relying on various versions of the Debreu and Scarf replicating technique consisting of expanding the demand side relative to each firm's production possibilities (see Debreu and Scarf, 1963). In our paper, no such replications are necessary, and the required level of competition is brought about by entry when entry costs are sufficiently small. However, while in the papers just mentioned existence of Cournot–Walras equilibria is shown indirectly when