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Journal of Mathematical Economics 41 (2005) 617–636

JOURNAL OF Mathematical ECONOMICS

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Existence and regularity of equilibria in a general equilibrium model with private provision of a public good

Antonio Villanacci[a,*], E. Ünal Zenginobuz[b]

[a] *Department of Mathematics for Decision Theory (Di.Ma.D), University of Florence, via Lombroso 6/17, 50134 Firenze, Italy*

[b] *Department of Economics, Boğaziçi University, 34342 Bebek, Istanbul, Turkey*

Received 14 May 2001; received in revised form 25 December 2004; accepted 31 December 2004
Available online 8 March 2005

Abstract

We prove existence and generic regularity of equilibria in a general equilibrium model of a completely decentralized pure public good economy. Competitive firms using private goods as inputs produce the public good, which is privately provided by households. Previous studies on private provision of public goods typically use one private good, one public good models in which the public good is produced through a constant returns to scale technology. As two distinguishing features of our model, we allow for the presence of *several* private goods and *nonlinear* production technology. In that framework, we use an homotopy argument to prove existence of equilibria and we show that economies are generically regular.

Keywords: General equilibrium model; Private provision of public good; Existence; Generic regularity

1. Introduction

In this paper, we prove the existence and regularity of equilibria in a general equilibrium model of a completely decentralized pure public good economy. The model studied extends

* Corresponding author. Tel.: +39 055 4796 817; fax: +39 055 4796 800.
E-mail addresses: villanac@unifi.it (Antonio Villanacci); zenginob@boun.edu.tr (E. Ünal Zenginobuz).

0304-4068/$ – see front matter
doi:10.1016/j.jmateco.2004.12.005

the standard pure exchange model with private goods by allowing households to make voluntary purchases of (or "privately provide") a public good that is produced by competitive firms using private goods as inputs.

The interest in a general equilibrium model with private provision of public goods lies in the fact that it serves as a benchmark extension of an analysis of completely decentralized private good economies to public good economies. Moreover, there are some relevant situations in which public goods are in fact privately provided, e.g., private donations to charity at a national and international level, campaign funds for political parties or special interests groups, and certain economic activities inside a family.

Previous studies on private provision of public goods typically use one private good, one public good models in which the public good is produced through a constant returns to scale technology. With only one private good, assuming constant returns to scale amounts to assuming linear production function, and as a result there is *no* loss of generality in normalizing prices of both the private and the public good to one. These assumptions also allow taking profits of firms equal to zero, with the implication that the presence of firms basically plays no role in the model. Therefore, as clearly explained by Bergstrom et al. (1986), the model reduces to a simple game where households are the players and the unique strategy variable is the choice of voluntary contribution. Then, proving existence is straight forward (see Bergstrom et al., 1986, Theorem 2, p. 33).[1]

As two distinguishing features of our model we have the presence of *several* private goods and we allow for *nonlinear* production technology for the public good. Therefore, the above-described price normalization will typically not be possible. Moreover, competitive firms will typically have non-zero profits, which are to be distributed among households. We are, therefore, analyzing a full-fledged general equilibrium model.

For given prices firms maximize profits. For given prices, initial endowments of private goods, ownership distribution of firms, as well as other households' choices of private provision of public goods, each household chooses a vector of private good consumption levels and a voluntary contribution level for the public good so as to maximize utility. Finally, markets for private and public goods clear. We show both existence and generic regularity of equilibria in this model.

We provide a proof of existence using a very simple homotopy argument, whose main characteristics consist in linking the true model with a fictitious one where the public good is treated as a private good. This strategy of proof turns out to be useful to show existence of equilibria in some related models as well.[2]

[1] The literature on voluntary contributions stems from Warr (1983), who studied the question of neutrality of taxes in a partial equilibrium public good model. Bergstrom et al. (1986) studied the same issue in a simple general equilibrium model with one private and one public good. The large literature that follows Bergstrom et al. (1986) mainly uses their framework and focuses on questions other than existence of equilibria in a full-fledged general equilibrium framework. A recent contribution by Cornes et al. (1999) on existence and uniqueness of equilibria in public good models also uses the framework in Bergstrom et al. (1986), i.e. a one private good, one public good economy with no relative price effects. The survey of papers on the existence of equilibria in private public good provision models by Cornes et al. (1999) confirms our observation that explicit work on existence of equilibrium in these models have indeed been limited to one public good, one private good case with no relative price effects.

[2] In a separate paper we prove, using the same approach, the existence of equilibria in a similar model where the public good is produced by a public firm subject to a budget constraint (Villanacci and Zenginobuz, 2004).