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A general equilibrium model of a production economy with asset markets

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Abstract

In this paper, a general equilibrium model of a monetary production economy is presented. The model is characterized by three classes of agents: a representative firm, heterogeneous households, and the government. Two markets (i.e., a labour market and a goods market, are considered) and two assets are traded in exchange of money, namely, government bonds and equities. Households provide the labour force and decide on consumption and savings, whereas the firm provides consumption goods and demands labour. The government receives taxes from households and pays interests on debt. The Walrasian equilibrium is derived analytically. The dynamics through quantity constrained equilibria out from the Walrasian equilibrium is also studied by means of computer simulations.

Keywords: Heterogeneous agents; General equilibrium; Financial markets and the macroeconomy; Computer simulations

0. Introduction

General equilibrium theory, developed by Léon Walras in the late 19th century and later extended to include uncertainty by Arrow and Debreu in the 1950s [1], investigates the existence of equilibrium levels of production, consumption and prices in a multi-market economy. General equilibrium theory is a milestone in modern economics. However, important and unsolved problems have been arisen by economists in the past and in the recent literature. Three important criticisms regard the lack of empirical validation, the tatonnement process and the fact that the model does not encompass money. In particular, monetary policy is in general neutral in a Walrasian general equilibrium model and this fact seems to contradict empirical evidence of real world economies characterized by decentralized exchange and price setting agents. Indeed, the framework of perfectly competitive markets with price-taking agents can be considered an useful benchmark in order to compare the welfare outcomes of more realistic models characterized by imperfect competition and price rigidities along the lines of new-Keynesian economics [2]. In this respect, this study presents a general equilibrium model that includes money and two asset markets, a bond market and a stock market. An experiment of expansionary monetary policy has been performed. The model can constitute a benchmark for further research on these topics.

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1. The model

1.1. Households

Each household i provides at each time step t the labour supply $L^s_{i,t}$ to the representative firm and demands consumption goods $C^d_{i,t}$, produced by the firm itself, according to an utility maximizing behaviour. Household's utility depends on current and expected leisure and on consumption stream. Leisure ℓ_i is defined as $\ell_i = L_{\max} - L_i$, where $L_{\max}$ is the maximum amount of working hours that household can provide at each time step. Households are myopic and are characterized by a two-period time-separable utility function, i.e.,

$$U_{i,t} = u(C_{i,t}, \ell_{i,t}) + E_{i,t}u(C_{i,t+1}, \ell_{i,t+1}), \tag{1}$$

where $E_{i,t}$ is the expected value at time t. For the sake of analytical tractability, a logarithmic elementary utility u is considered. Furthermore, households suppose to be unemployed at time $t+1$. Hence, each household i sets the schedules of labour supply L^s_i and consumption demand C^d_i by maximizing its intertemporal utility subject to a budget constraint. The problem can thus be stated as follows:

$$\max_{C^d_{i,t}, L^s_{i,t}} U_{i,t} = a \log C^d_{i,t} + c \log\left(1 - \frac{L^s_{i,t}}{L_{\max}}\right) + bE_{i,t} \log\left(\frac{\hat{W}_{i,t+1}}{p_{t+1}}\right), \tag{2}$$

with

$$\hat{W}_{i,t+1} = (1 + \rho_{i,t+1})\hat{\Pi}_{i,t}, \tag{3}$$

$$\hat{\Pi}_{i,t} = W_{i,t} - p_t C^d_{i,t} + w_t L^s_{i,t} + d_t S_{i,t} + r_t B_{i,t} - T_{i,t}, \tag{4}$$

where $W_t = \sum_i W_{i,t}$, $S_t = \sum_i S_{i,t}$, $B_t = \sum_i B_{i,t}$, $T_t = \sum_i T_{i,t}$. Parameters a, c and b give the relative weights of the utility of present consumption, leisure, and of expected utility of real wealth in the next period, respectively. Eq. (4) represents the budget constraint, where $W_{i,t}$ is the current nominal wealth, $p_t C^d_{i,t}$ is the money that should be paid for buying the desired amount of goods, $w_t L^s_{i,t}$ is the salary that would be received for the desired labour supply, $d_t S_{i,t}$ and $r_t B_{i,t}$ are the capital income, i.e., dividends and interests paid by equities and government bonds held by household i at time t. $T_{i,t}$ are the nominal amount of taxes paid to the government. The real level of wealth expected at time $t+1$, i.e., $\hat{W}_{i,t+1}/p_{t+1}$, is taken as a proxy of consumption expected at the same time. The price level p_t, the nominal wage w_t, the interest rate r_t and the aggregate amount of dividends d_t are known to all households at the beginning of time step t. Households act as price takers in all markets. It is worth noting that, out of the Walrasian market clearing equilibrium, households may be rationed in their transactions. In this case, $C^d_{i,t}$ and $L^s_{i,t}$ would not represent the effective amounts of goods and labour traded at time t. However, each household is supposed to behave without considering possible rationing and considers only notional demand and supply schedules, $C^d_{i,t}$ and $L^s_{i,t}$, for its wealth dynamics. Eq. (3) and (4) give the expected dynamics of wealth stated that $\hat{\Pi}_{i,t}$ is the money amount reinvested at time t in the bond market and in the stock market at the unknown stochastic rate $\rho_{i,t+1}$. The solution of the problem stated by Eqs. (2)–(4) gives the notional schedules for goods demand $C^d_{i,t}$ and labour's supply $L^s_{i,t}$ for each household:

$$C^d_{i,t} = \frac{a}{a+b+c}\frac{w_t}{p_t}L_{\max} + \frac{a}{a+b+c}\frac{W_{i,t} + r_t B_{i,t} + d_t S_{i,t} - T_{i,t}}{p_t}, \tag{5}$$

$$L^s_{i,t} = \frac{a+b}{a+b+c}L_{\max} + \frac{c}{a+b+c}\frac{W_{i,t} + r_t B_{i,t} + d_t S_{i,t} - T_{i,t}}{w_t}. \tag{6}$$

After transactions take place in the goods and labour markets, asset markets open. Each household i allocates the amount $\Pi_i = W_{i,t} - p_t C_{i,t} + w_t L_{i,t} + d_t S_{i,t} + r_t B_{i,t} - T_{i,t}$ in stocks and government bonds according to Markowitz's portfolio theory. $C_{i,t}$ and $L_{i,t}$ are the effective amounts of goods and labour traded by household i in their respective markets. Their values are set at the minimum between aggregate demand and supply. Uniform rationing has been applied, see Ref. [3, Chapter 2].