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ScienceDirect
Journal of Econometrics 136 (2007) 699–723
JOURNAL OF Econometrics
www.elsevier.com/locate/jeconom

# Evaluation of dynamic stochastic general equilibrium models based on distributional comparison of simulated and historical data

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Available online 7 February 2006

**Abstract**

We take as a starting point the existence of a joint distribution implied by different dynamic stochastic general equilibrium (DSGE) models, all of which are potentially misspecified. Our objective is to compare "true" joint distributions with ones generated by given DSGEs. This is accomplished via comparison of the empirical joint distributions (or confidence intervals) of historical and simulated time series. The tool draws on recent advances in the theory of the bootstrap, Kolmogorov type testing, and other work on the evaluation of DSGEs, aimed at comparing the second order properties of historical and simulated time series. We begin by fixing a given model as the "benchmark" model, against which all "alternative" models are to be compared. We then test whether at least one of the alternative models provides a more "accurate" approximation to the true cumulative distribution than does the benchmark model, where accuracy is measured in terms of distributional square error. Bootstrap critical values are discussed, and an illustrative example is given, in which it is shown that alternative versions of a standard DSGE model in which calibrated parameters are allowed to vary slightly perform equally well. On the other hand, there are stark differences between models when the shocks driving the models are assigned non-plausible variances and/or distributional assumptions.


*JEL classification:* C12; C22

*Keywords:* Real business cycles; Output; Empirical distribution; Simulated models; Model selection

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0304-4076/$ - see front matter 
doi:10.1016/j.jeconom.2005.11.010

## 1. Introduction

In this paper, we merge recent econometric advances in bootstrapping time series and Kolmogorov type testing with recent developments in the evaluation of dynamic stochastic general equilibrium (DSGE) models. This is accomplished via the construction of a new tool for comparing the empirical joint distribution of historical time series with the empirical distribution of simulated time series.

Since the seminal papers by Kydland and Prescott (1982), Long and Plosser (1983) and King et al. (1988a,b), there has been substantial attention given to the problem of reconciling the dynamic properties of data simulated from DSGE, and in particular from real business cycle (RBC) models, with the historical record. A partial list of advances in this area includes: (i) the examination of how RBC simulated data reproduce the covariance and autocorrelation functions of actual time series (see e.g. Watson, 1993); (ii) the comparison of DSGE and historical spectral densities (see e.g. Diebold et al., 1998a); (iii) the evaluation of the difference between the second order time series properties of vector autoregression (VAR) predictions and out-of-sample predictions from DSGE models (see e.g. Schmitt-Grohe, 2000); (iv) the construction of Bayesian odds ratios for comparing DSGE models with unrestricted VAR models (see e.g. Chang et al., 2002, and Fernandez-Villaverde and Rubio-Ramirez, 2004); (v) the comparison of historical and simulated data impulse response functions (see e.g. Cogley and Nason, 1995); (vi) the formulation of "reality" bounds for measuring how close the density of a DSGE model is to the density associated with an unrestricted VAR model (see e.g. Bierens and Swanson, 2000); and (vii) loss function based evaluation of DSGE models (Schorfheide, 2000). The papers listed above are mainly concerned with the issue of model evaluation. Another strand of the literature is instead mainly concerned with providing alternatives to calibration (see e.g. DeJong et al., 2000 for a Bayesian perspective in which prior distributions are constructed around calibrated structural parameters). In most of the above papers, the issue of singularity (i.e. when the number of variables in the model is larger than the number of shocks) is circumvented by considering only a subset of variables, for which a non-degenerate distribution exists.[1] Our work is closest to the first strand of literature. In particular, our paper attempts to add to the model evaluation literature by introducing a measure of the "goodness of fit" of RBC models that is based on applying standard notions of Kolmogorov distance and drawing on advances in the theory of the bootstrap.[2]

The papers cited above primarily address the case in which the objective is to test for the correct specification of some aspects of a given candidate model. In the case of DSGE

[1] A novel alternative to calibration is proposed by Bierens (2005). He solves the singularity problem via convolution of the singular distribution of an RBC model with a non-singular distribution. The same convolution is also applied to the associated non-singular distribution of the econometric model, which is an unrestricted VAR. Parameters are then estimated via the maximization of an information criterion which gives the probability that the distribution of the convoluted VAR model is generated by the distribution of the convoluted RBC model, conditional on the data.

[2] In recent years, Kolmogorov type distance measures for testing distributional equality have been extended to the case of dependent observations and/or parameter estimation error (see e.g. Andrews, 1997; Bai, 2003). Tests of this sort generally have limiting distributions that are not nuisance parameter free, and critical values cannot be tabulated. Papers addressing this issue in the bootstrap literature include Andrews (2002), Goncalves and White (2004), Hall and Horowitz (1996), Horowitz (2003), Inoue and Shintani (2001) and Naik-Nimbalkar and Rajarshi (1994).