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A reduced order system ID approach to the modelling of nonlinear structural behavior in aeroelasticity

P.J. Attar[a,*], E.H. Dowell[b,1]

[a]Air Force Research Laboratory, AFRL/VAAC, 2210 Eighth St., Bldg 146, WPAFB,OH 45433-7512, USA
[b]Pratt School of Engineering, Box 90300, Duke University, Durham NC 27708, USA

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Abstract

A method is proposed for identifying a set of reduced order, nonlinear equations which describe the structural behavior of aeroelastic configurations. The strain energy of the system is written as a (polynomial) function of the structures' modal amplitudes. The unknown coefficients of these polynomials are then computed using the strain energy data calculated from a steady state, high-order, nonlinear finite element model. The resulting strain energy expression can then be used to develop the modal equations of motion. From these equations, zero and nonzero angle of attack flutter and limit cycle oscillation data are computed for a 45° delta wing aeroelastic model. The results computed using the reduced order model compare well with those from a high-fidelity aeroelastic model and to experiment. A two to three order of magnitude reduction in the number of structural equations and a two order of magnitude reduction in total computational time is accomplished using the current reduced order method.

Keywords: Nonlinear aeroelasticity; System identification

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1. Introduction

In recent years various reduced order methods have been developed and used in the field of aeroelasticity. Normal mode reduced order modal methods, which have been used in the field of structural dynamics for many years, have now also been applied to obtain reduced order aerodynamic models (Dowell, 1996; Hall, 1994; Tang et al., 1999). In cases where the computation of the normal modes of the structural or aerodynamic system becomes too computationally expensive, methods such as the proper orthogonal decomposition technique (POD), which extract dynamic information from time domain or frequency domain computations, have been utilized (Epureanu et al., 2000, 2001; Hall et al., 1999; Romanowski, 1996). Also Hall and his colleagues have recently developed another novel technique which uses a harmonic balance solution for a set of nonlinear ordinary equations in time. This method has been used to compute solutions to nonlinear computational fluid dynamics (CFD) problems (Hall et al., 2002; Thomas et al., 2002, 2004; Kholodar et al., 2002).

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*Corresponding author. Tel.: +1 937 255 0057; fax: +1 937 656 7857.
E-mail addresses: peter.attar@wpafb.af.mil (P.J. Attar), dowell@ee.duke.edu (E.H. Dowell).

[1] William Holland Hall Professor, Department of Mechanical Engineering and Materials Science, Director of the Center for Nonlinear and Complex Systems and Dean Emeritus, Duke University.

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Nomenclature

$\boldsymbol{A_b}$	matrix of aerodynamic bound circulation influence coefficients
$\boldsymbol{A_w}$	matrix of aerodynamic wake circulation influence coefficients
$A_{ijkl\cdots o}$, A_i	polynomial coefficients in strain energy expression
$\boldsymbol{C}$	structural damping matrix
$\boldsymbol{E}$	matrix of interpolation coefficients
$\boldsymbol{F}^P$	vector of structural nodal forces due to applied pressure
$\boldsymbol{F}^{SE}$	vector of structural nodal forces derived from a strain energy function
$\tilde{\boldsymbol{F}}^{SE}$	vector of structural modal forces derived from a strain energy function
$\boldsymbol{M}$	structural mass matrix
$\boldsymbol{M}_i$	largest power to which ith modal amplitude is raised to in strain energy expression
$\boldsymbol{N}$	number of structural modes kept in modal expansion
$\boldsymbol{n}$	discrete time level in aerodynamic equations of motion
$\boldsymbol{N}_t$	number of terms in strain energy expression
$\boldsymbol{N}_{\text{full}}$	number of degrees of freedom in finite element model
$\boldsymbol{q}$	vector of structural modal amplitudes
U	strain energy function
$\boldsymbol{u}_z$	vector of out-of-plane nodal degrees of freedom
$\boldsymbol{u}$	vector of structural nodal degrees of freedom
$\boldsymbol{\alpha}$	wing angle of attack
$\boldsymbol{\Gamma_b}$	vector of aerodynamic bound circulation
$\boldsymbol{\Gamma_w}$	vector of aerodynamic wake circulation
$\boldsymbol{\Phi}$	matrix of structural modal vectors
$\boldsymbol{\phi}_i$	ith structural modal vector
$\psi_i(q_1, q_2, \ldots, q_N)$	the ith polynomial basis function in the strain energy expression

A disadvantage of the methods used in the above studies is that the original full order equations of motion must be available to construct the reduced order model. If, for example, the engineer wants to construct a reduced order model for a system which was initially modelled using a commercial package, then most often this approach will not be viable since the actual equations are not available to the user. Worst yet, the situation could be that the engineer has data which was taken experimentally and for which a theoretical model has not been constructed. In these cases a method must be used which can take output and/or input data from a "black box" source and construct a reduced set of equations which can be used as a predictive tool. These methods are often called system identification techniques.

In the field of structural dynamics, a large amount of research exists on identifying the parameters for linear systems. Juang (1994) and Juang and Pappa (1985) have developed techniques which are able to take input and output data and construct the system modal damping and frequencies. These same methods were used to determine the controllability of aerodynamic modes in a linear vortex lattice aerodynamic model (Tang et al., 2001; Kim, 2004). However the methods used in the work by Juang and Pappa (1985) and Kim (2004) will only be valid if the considered system can be assumed to be behaving in a linear manner.

In recent years some work has been done in the area of nonlinear system identification. In recent work by Epureanu and Dowell (2003), a multivariate, third order polynomial was used as the functional form for a nonlinear structural model used in the computation of panel limit cycle oscillations (LCO). Time series data from a finite difference solution of the original nonlinear equations was used to identify the coefficients of the polynomial. Gabbay et al. (2000) and Mehner et al. (2000) used polynomial functions whose coefficients were identified using strain energy data from a series of finite element runs, to model a microelectromechanical system. Silva et al. have used a Volterra series approach to identify linear and nonlinear aerodynamic systems (Silva, 1997, 1999; Silva and Raveh, 2001; Silva and Bartels, 2002; Lucia et al., 2003). Denegri and Johnson (2001) have conducted a study based upon a neural network model using flight test data which has shown promise. Lucia et al. (2004) have prepared a comprehensive review of reduced order modelling methods including those based upon system identification.

In the work to be presented here, the LCO of a 45° delta wing will be computed using a linear vortex lattice model which is coupled to a nonlinear structural model. The structural model used here is constructed by a nonlinear system identification of a high-fidelity nonlinear finite element model. Flutter and LCO results are presented for zero and nonzero angles of attack. The results from this model are compared to results for two different theoretical models and to experimental results.

2. Theory

2.1. Full aeroelastic equations

In the work to be presented in this paper, the nonlinear aeroelastic model consists of a linear vortex lattice aerodynamic model coupled to a nonlinear structural model. Only an abbreviated discussion of the full order system of