Available online at www.sciencedirect.com

ScienceDirect

Composite Structures 82 (2008) 140–154

COMPOSITE STRUCTURES

www.elsevier.com/locate/compstruct

Enhanced structural behavior of flexible laminated composite beams

Yulia Fridman, Haim Abramovich *

Faculty of Aerospace Engineering, Technion IIT, 32000 Haifa, Israel

Abstract

The aim of the present study is to investigate analytically and numerically the structural behavior of laminated composite beams under axial compression using piezoelectric layers. A mathematical model was developed based on a first order shear deformation theory (FSDT) which includes shear deformations, usually neglected in the classical lamination theory of composite structures. Closed form solutions for the bending angle and the axial and lateral displacements along the beam are presented for various boundary conditions.

Natural frequencies and their associated mode shapes, as well as, buckling loads were computed for beams with and without piezoelectric layers influence, having various boundary conditions and lay-ups.

Next, the influence of the piezoelectric layers on the axial compression load and the natural frequencies is investigated to yield an enhancement of the structural behavior of the beam. This is done using a proportional control load, in which the sensed voltage on the beam is fed back (after being amplified using a constant gain G), onto the PZT actuators which prevent the premature buckling of the flexible beam by actively increasing its stiffness.

A parametric investigation was performed for beams with various lay-ups, and it was shown that for a given feedback gain value, the natural frequencies and the buckling loads can be increased by a factor of two, when using the enhancement procedure developed within the present study.

Keywords: Laminated composite beam; PZT; Buckling load; Gain; Enhanced buckling load; Piezoelectric patch

1. Introduction

The issue of increasing the buckling loads and natural frequencies of beams and plates using intelligent materials such as piezoelectric or shape memory alloy (SMA) materials had recently become very popular.

However most of the studies concentrate on basic models, like Euler–Bernoulli beam theory, while most of the emphasis is made on the control part. Also most of the works had only focused on the control of structural vibrations. Typical references are next highlighted.

Abramovich and Livshits [1] investigated the dynamical behavior of piezo-laminated composite beams with general non-symmetric lay-up. The natural frequencies of the beam with various boundary conditions were calculated and the influence of continuous piezoelectric layers bonded on the top and bottom of the beams, acting as actuators in the open loop, was investigated. Song et al. [2] studied the active vibration control of composite beams using piezoelectric ceramic patches as sensors and actuators. Two control algorithms were developed to achieve the active vibration damping. The results were calculated based on the theoretical Euler–Bernoulli model, and numerical simulations were obtained using the ANSYS finite element code. A good correlation was shown between the numerical predictions and the experimental results. Huang and Sun [3] developed a beam model based on the Reissner–Mindlin plate theory to demonstrate the dynamic analysis of composite beams with bonded or embedded composite sensors and actuators. Waisman and Abramovich [4] studied the stiffening effects of smart composite beams with piezo-ceramics layers or patches bonded on the surface of the beam. The analysis considers the linear piezoelectric constitutive relations and the first

---

* Corresponding author. Tel.: +972 4 8293199; fax: +972 4 8292303.
E-mail addresses: fyulia@tx.technion.ac.il (Y. Fridman), haim@aerodyne.technion.ac.il (H. Abramovich).

0263-8223/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compstruct.2007.05.007

order shear deformation theory. The influence of the actuators is evaluated by means of the pin-force model and their size and location along the beam are taken into account. The numerical solution of the equations of motion are compared with the FE results and it was found that piezoelectric bonded actuators are yielding significant changes in the natural frequencies and mode shapes of the beam. Raja et al. [5] derived a finite element formulation capable of modeling extension–bending and shear induced actuation in an adaptive composite sandwich beam based on Timoshenko's beam theory. The efficiency of both actuators in controlling the bending vibration was studied using modal control analysis. The shear actuator performs better in controlling the first three bending modes than the extension-bending actuator. Sloss et al. [6] investigated the effect of axial load, the piezo-sensor and the actuator feedback control on the vibration frequencies and the mode shapes of the beam. The first three vibration frequencies of controlled and uncontrolled beam were presented for various values of the feedback gain, axial load and patch sizes. Karagulle et al. [7] showed that the Ansys/Multiphysics finite element code can be used for the simulation of the active vibration control of smart structures.

Mukherjee and Chaudhuri [8] presented an exact solution for the feedback vibration control of piezo-laminated columns, in which the signals from the sensors attached to the structure are fed back to the actuators with a gain multiplier. The analytical solution has been validated with experimental studies for tip loaded piezo-laminated cantilever beam with collocated PVDF layers using as sensors and as actuators.

In their previous work, Mukherjee and Chaudhuri [9] developed an imperfection approach for exact solutions of the instability of piezo-laminated symmetrical columns under static and dynamics axial loads. Their solution is based on Euler–Bernoulli beam theory. A constant gain feedback control algorithm is derived using modified stiffness yielding an increased Euler's buckling load while using piezoelectric sensing and actuation. A limiting actuation feedback gain is derived. Meressi and Paden [10] observed that the buckling of a flexible Euler–Bernoulli beam can be postponed beyond the first and under the second critical load by stabilization of the first bending mode by means of a feedback control using piezoelectric actuators and strain sensors. This is followed by the state-space model of the reduced order system and designing of a controller by using standard linear quadratic regulator (LQR) with constant feedback gain. Chase et al. presented optimal stabilizations of column buckling [11] and plate buckling [12] using MEMS-based strain sensors and embedded piezoelectric ceramic patches. The column is fixed with pinned ends and the axial load is applied dynamically. The stability of the resulting controllers is based on multi-input, multi-output (MIMO) methodology using Lyapunov's methods. The finite element column model is presented in state-space equations. The optimal buckling controllers were tested on a column made of G-10 fiberglass yielding an increase in the critical buckling load by a factor of 2.9. In Ref. [12] the derivation of Ref. [11] is expanded to the case of axially loaded composite plates clamped on all four sides. The controller is designed for $P_{\mathrm{desired}} = 1.5P_{\mathrm{cr}}$. For this case, the calculations became complicate with a high requirement for computer time and control power.

Thompson and Loughlan [13] performed experiments on the active buckling control of pin-ended composite column strips made of graphite-epoxy using lateral deflection displacement sensor and surface bonded piezo-ceramic actuators. The test procedure is outlined and load–deflection plots, obtained with and without active control, are presented. For the lay-up configurations considered, the increase in the load carrying capability is of the order of 19.8–37.1%.

Chandrashekhara and Bhatia [14] presented a finite element analysis for active buckling control of laminated composite plates using piezoelectric sensors and actuators. The finite element model is based on the first order shear deformation plate theory in conjunction with linear piezoelectric theory. The sensor output is used to determine the input to the actuator using proportional control algorithm. The presented finite element solutions show effectiveness of piezoelectric materials in enhancing the buckling loads. Berlin [15] presented results of an analysis based on Euler–Bernoulli beam theory together with an experimental work and showed that buckling can be prevented trough computer-controlled adjustment of dynamical behavior. He used piezo-ceramic actuators bonded on the surface of a steel column to counteract buckling. Active control of the buckling allows this column to support 5.6 times more axial load then the original buckling load. This was done with a complicated control law mechanism. Wang and Quek [16] showed the increase of the flutter velocity and buckling capacity of a fixed-free column, subjected to a follower force using a pair of piezoelectric layers.

In the present study, a mathematical model of a piezo-laminated composite beam was developed, formulated and applied. It is based on a first order shear deformation theory (FSDT) which includes shear deformations, usually neglected in classical lamination theory of composite structures, and linear piezoelectric constitutive relations. The three coupled partial differential equations of motion of a general non-symmetric piezo-laminated composite beam subjected to axial and lateral tractions are presented and solved for the dynamic case – to find natural frequencies and mode shapes for an axially compressed beam, with and without axially loads, and for the static case – to find the buckling load, the bending angle, the axial and lateral displacements.

Moreover, the buckling load of a laminated composite beam with various boundary conditions is enhanced using piezoelectric sensors and actuators. Various lay-ups are considered, including symmetric and non-symmetric ones. The enhanced buckling loads are shown to be twice the