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# Detection and quantification of non-linear structural behavior using principal component analysis

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ABSTRACT

The detection of non-linear behavior in structural dynamics is a very important step to the extent that the presence of non-linearities, even local, can affect the global dynamic behavior of a structure. A large number of techniques that enable engineers to detect non-linear behavior can be found in the literature but most of these methods exploit frequency domain data and give better results with a stepped-sine excitation. The goal of this paper is to propose an alternative methodology that is based on the principal component analysis and uses time responses obtained with a random excitation. Two criteria will be used to quantify the difference between two response subspaces, based on the angle between them and the residual error resulting from the projection of one on the other. The concept of limit of linearity and design decision margins is also addressed in this paper. The methodology is demonstrated using an academic simulated system and then using measured data of a simplified solar array system.



## 1. Introduction

Structural dynamic behavior must generally be taken into account in the design of mechanical systems in order to insure their performance and reliability. Confrontation between numerical simulations and experimental observations on a prototype often indicates that the model is unsatisfactory and model validation strategies in general [1] and model updating methodologies in particular [2] have been developed over the years to support the engineer in improving model fidelity. However, most of these strategies are restricted to linear elastodynamic behaviors. Indeed, while many industrial structures are inherently non-linear, the difficulty of modeling these effects in structural dynamics and the computational burden of performing the non-linear calculations have discouraged engineers in the past from performing non-linear simulations. However, the increasing need for high-fidelity simulations and the availability of commercial non-linear simulation software have motivated the development of new approaches for validating non-linear structural systems.

Non-linear system identification is an integral part of the validation process, and it can be viewed as a succession of three steps: detection, characterization and parameter estimation. This paper focuses on the detection step that enables to know whether or not the tested structure has a significant level of non-linear behavior and whether or not it can be safely neglected.

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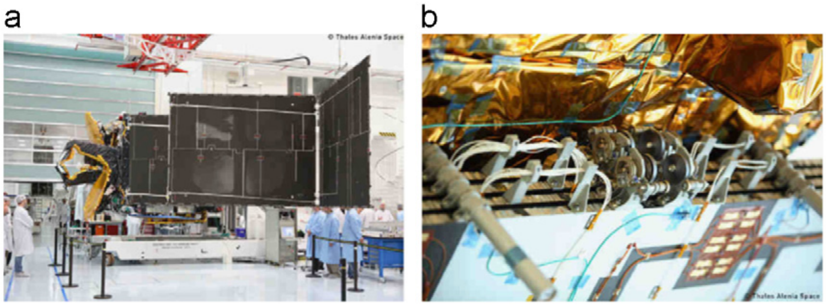

**Fig. 1.** Views on solar generators. (a) Opening test and (b) complex connexions between panels.

A specific difficulty encountered with space structures is the behavior of solar generators in their stowed position. Indeed, these solar panels are folded during the spacecraft launch and impact one another at specific points. Interface conditions between the different panels, presented in Fig. 1(b), are also sources of non-linear behaviors: friction and gaps may appear in clamping as the excitation level increases. These are just some of the many conditions which lead to non-linear phenomena and that engineers would like to detect.

Several methods that have proved to be useful for detecting non-linearity can be found in the literature. The most well-known techniques use frequency domain data. For example, the homogeneity test examines distortions in the Frequency Response Functions (FRF) for several levels of excitation; the detection of jump phenomenon in the FRF due to the non-uniqueness of the solutions for a non-linear system; the Hilbert transform that differs from the original FRF. Readers can refer to [3,4] for more details on these detection methods. Moreover, the results of these methodologies depend on the excitation type. The stepped-sine excitation gives the best results since it enables to obtain a well-defined FRF where the distortion clearly appears. But, this type of excitation involves a long test duration and that is why the most commonly used excitations are either random or swept-sine. In [5], the authors use these detection techniques and observe distorted linearity plots obtained by ground vibration test on aircraft for some typical non-linear phenomena. A novel detection approach, based on the concept of non-linear output frequency response functions (NOFRF), is proposed in [6], and the authors show that they are able to localize the unknown non-linear element in a 10 dof system.

Fewer techniques exist that use only time data. For example, in [7] the Hilbert transform is applied to signals in the time domain in order to extract the instantaneous dynamic characteristics of the structure. Then, using the time varying envelope as well as the instantaneous phase and frequency, the authors are able to detect a non-linear behavior. The continuous wavelet transform used in [8] is a method that uses the free response of a non-linear system. The authors detect non-linearities by looking at distortions in the amplitude and the phase of the wavelet.

The aim of the study presented in this paper is to propose an alternative methodology based on Principal Component Analysis (PCA) in order to detect a non-linear behavior. This approach has three main advantages. First, it uses only time domain data so that no signal processing transformation of the measurements is needed, and all the information are conserved. Second, the type of excitation used here is random, which is generally not chosen because of its poor results in the frequency domain. Third, a limit of linearity will be computed and will help to state whether or not the structure is non-linear.

## 2. Principal component analysis

### 2.1. Theoretical background

Principal Component Analysis is a statistical multivariate analysis technique whose goal is to reduce the dimension of a response matrix and to retain the dominant information in the data. PCA is closely related to Singular Value Decomposition (SVD) and Proper Orthogonal Decomposition, also known as Karhunen–Loève decomposition. One specific application of the PCA in the field of structural dynamics is to find the subspaces spanned by the principal directions that contain most of the system's energy, without calculating the modes shapes.

Given a response matrix $X$ containing the displacements $x_i(t_j)$ obtained over $n$ sensors on the studied structure during $m$ time samples:

$$X = \begin{pmatrix} x_1(t_1) & \dots & x_1(t_m) \\ \vdots & \ddots & \vdots \\ x_n(t_1) & \dots & x_n(t_m) \end{pmatrix} \tag{1}$$

The SVD is then calculated in order to obtain the principal directions:

$$X = U\Sigma V^T \tag{2}$$