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Decision-theoretic specification of credal networks: A unified language for uncertain modeling with sets of Bayesian networks

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Abstract

Credal networks are models that extend Bayesian nets to deal with imprecision in probability, and can actually be regarded as sets of Bayesian nets. Credal nets appear to be powerful means to represent and deal with many important and challenging problems in uncertain reasoning. We give examples to show that some of these problems can only be modeled by credal nets called *non-separately specified*. These, however, are still missing a graphical representation language and updating algorithms. The situation is quite the opposite with separately specified credal nets, which have been the subject of much study and algorithmic development. This paper gives two major contributions. First, it delivers a new graphical language to formulate any type of credal network, both separately and non-separately specified. Second, it shows that *any* non-separately specified net represented with the new language can be easily transformed into an equivalent separately specified net, defined over a larger domain. This result opens up a number of new outlooks and concrete outcomes: first of all, it immediately enables the existing algorithms for separately specified credal nets to be applied to non-separately specified ones. We explore this possibility for the 2U algorithm: an algorithm for exact updating of singly connected credal nets, which is extended by our results to a class of non-separately specified models. We also consider the problem of inference on Bayesian networks, when the reason that prevents some of the variables from being observed is unknown. The problem is first reformulated in the new graphical language, and then mapped into an equivalent problem on a separately specified net. This provides a first algorithmic approach to this kind of inference, which is also proved to be NP-hard by similar transformations based on our formalism.

Keywords: Probabilistic graphical models; Bayesian networks; Credal networks; Credal sets; Imprecise probabilities; Conservative updating; Conservative inference rule

1. Introduction

We focus on credal networks (Section 3) [1], which are a generalization of Bayesian nets. The generalization is achieved by relaxing the requirement that the conditional mass functions of the model be precise: with credal

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nets each of them is only required to belong to a closed convex set. Closed convex sets of mass functions are also known as *credal sets* after Levi [2]. Using credal sets in the place of mass functions makes credal networks an *imprecise probability* model [3]. It can be shown, in particular, that a credal network is equivalent to a *set of Bayesian nets* with the same graph.

An important question is whether or not all credal networks can be represented in a way that emphasizes locality. The answer is positive if we restrict the attention to the most popular type of credal networks, those called *separately specified* (Section 4). In this case, each conditional mass function is allowed to vary in its credal set independently of the others. The representation is naturally local because there are no relationships between different credal sets. The question is more complicated with more general specifications of credal networks, which we call *non-separately specified* (Section 5). The idea of non-separately specified credal nets is in fact to allow for relationships between conditional mass functions in different credal sets, which can be far away from each other in the net.

Although the idea of non-separately specified credal nets is relatively intuitive, it should be stressed that this kind of nets has been investigated very little: in fact, there has been no attempt so far to develop a general graphical language to describe them; and there is no algorithm to compute with them.[1] This appears to be an unfortunate gap in the literature as the non-separate specification seems to be the key to model many important problems (Sections 5.1–5.4). Separately specified credal nets, on the other hand, have been the subject of much algorithmic development (see [1] for an overview of the main results in this field).

In this paper, we give two major contributions. First, we define a unified graphical language to locally specify credal networks in the general case (Section 6). This specification is called *decision-theoretic* being inspired, via the Cano-Cano-Moral (CCM) transformation [5], by the formalism of influence diagrams, and more generally of decision graphs [6]. In this language the graph of a credal network is augmented with control nodes that express the relationships between different credal sets. We give examples to show that the new language provides one with a natural way to define non-separately specified nets; and we give a procedure to reformulate any separately specified net in the new language.

Second, we make a very simple observation (Section 7), which has surprisingly powerful implications: we show that for any credal network specified with the new language there is a separately specified credal network, defined over a larger domain, which is equivalent. The procedure to transform the former into the latter network is very simple, and takes only linear time. The key point is that this procedure can be used as a tool to "separate" the credal sets of non-separately specified nets. This makes it possible to model, by separately specified nets, problems formerly modeled by non-separately specified ones; and hence to use *any* (both exact and approximate) existing algorithm for separately specified nets to solve such problems.

In Section 8, we explore this possibility in the case of the *2U algorithm* [7]: a polynomial time algorithm for exact updating of singly connected credal networks with binary variables. We show that the algorithm, originally designed only for separately specified credal networks, can be extended to deal exactly and efficiently also with a class of non-separately specified models.

Our contributions also apply to the problem of belief updating on Bayesian networks by the *conservative inference rule* [8], which is a rule modeling situations where the reason that prevents some of the variables from being observed is unknown. The problem has been mapped onto a standard updating problem on a non-separately specified credal network [9], a result not straightforward to exploit in practice because of the lack of algorithms for non-separately specified credal networks. A feasible solution of this problem based on our formalism is presented in Section 9. First, we represent the problem by the new decision-theoretic language. Second, we use our transformation to reformulate the problem on a separately specified credal network defined over a larger domain. At this point, the problem can be solved by the existing algorithms for separately specified credal nets. Additionally, we also prove the NP-hardness of belief updating with this rule by similar transformations based on the results presented in this paper.

Some comments and perspectives for future developments are discussed in Section 10. The more technical parts of this paper are collected in Appendix A.

[1] An exception is the classification algorithm developed for the *naive credal classifier* [4], but it is ad hoc for a very specific type of network. More generally speaking, it is not unlikely that some of the existing algorithms for separately specified nets can be extended to special cases of non-separate specification, but we are not aware of any published work dealing with this issue.